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Automatic Discovery of Heuristics for Nondeterministic Programs from Sample Execution Traces

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CHAPTER 1. INTRODUCTION

1.1 Background and Scope of Thesis

During the last few years, a number of relatively effective artificial intelligence (AI) programs have been written incorporating considerable amounts of knowledge. Consequently, the problem of encoding such knowledge in a useful form has emerged as one of the central problems of AI. Winograd [48] distinguishes between *declarative* information which can be thought of as "knowing what", and *procedural* information which can be thought of as "knowing how". He describes the underlying problem as that of constructing representations which can take advantage of the decomposability of the declarative form without sacrificing the interactive possibilities of the procedural form. Ideally, it should be possible to specify information in a form which does not constrain the way in which it is to be used. Unfortunately, a straightforward implementation of such a declarative representation corresponds to a nondeterministic program which makes a relatively blind search through the solution space.

More recently, attention has turned towards mechanisms which facilitate incorporating limited procedural or heuristic information into a primarily declarative framework. Examples

include Sickel's Clause Interconnectivity Graphs [39] in a resolution theorem-proving framework, Robot Plans in the STRIPS problem-solving system [14], and procedural deductions in semantic networks [10]. In [36], Rychener discusses the approach of building a rational "goal" structure into declarative rule-based (production) systems, while Davis [9] favors a separate set of "meta-rules" specifying control information in a hierarchical fashion. In some cases we might expect that the amount of procedural information is of the same order of magnitude as the amount of declarative information, and so such approaches may be effective. However, in most cases the amount of control information will be very large (such as theorem-proving, natural language understanding, vision). Thus the problem of acquiring, debugging and extending control information will become increasingly important.

In general, we might expect that this control information will embody very sophisticated principles requiring data structures not present in the declarative form of the program, or which are deducible only by the use of considerable intelligence. For example, it is clear that the deduction of the heuristic principles of evaluation and α-β search from a declarative chess program which specifies only the rules of chess without any strategies for playing will require an analysis which is considerably beyond the ability of present techniques; even the optimization of the

parameters in a linear evaluation function involves highly sophisticated processing.

However, it is the contention of this thesis that there are a number of important areas in which it might be possible to deduce control information automatically from a declarative program. These include declaratively specified problems:

- for which there exists a relatively simple algorithmic procedure
- whose performance can be improved in frequently occurring or particularly important special cases
- in which particular subproblems can be solved by simple algorithmic procedures.

In this thesis, we will investigate a technique for improving the performance of a nondeterministic program which is based on an analysis of the behavior of the program on sample inputs.

The following section presents a sample problem used throughout this document to demonstrate the proposed techniques. In Section 1.3, the approach we take to this problem is outlined in detail along with the technical problems involved in the actual analysis. The final section of this chapter presents a history of the research related to this problem and which supports the approach we use.

1.2 A Sample Problem: Jigsaw Puzzles

As an example, consider the following problem. Suppose we want to solve a (slightly idealized) jigsaw puzzle in which each piece has an average color and four sides described by unique integers (with side i fitting side -i). We consider below a nondeterministic algorithm written in *Production System* (PS) form [8,32,37].

A PS is a (nondeterministic) program consisting of a set of *productions* or *rules*, called *Production Memory* together with a data base of assertions, called the *Working Memory* (WM) set. Each rule consists of a conjunction of patterns of data elements, called the *Left-hand Side* (LHS) and a series of actions called the *Right-hand Side* (RHS). The RHS specifies information that is to be deposited in or removed from the WM. Execution of the program consists of iterating the following sequence of actions (each iteration is called a *cycle*):

(1) for each rule, determine if its LHS matches the current environment in WM (multiple instantiations are possible)

(2) from the set of rules satisfying step (1), called the *Conflict Set* (CS), nondeterministically select one.

(3) *fire* the rule selected in step (2), that is, apply the actions specified in the RHS.

In the exact form of the PS representation we use, data elements can be any LISP data structure. An atomic data element in the LHS of a production must match an exact

data element in WM and a list must match a list with the same structure and content. A symbol preceded with an equals sign (=) represents a variable which can match any data structure. The $ symbol contained in the productions has the same function as the SNOBOL4 immediate assignment operator. When a rule is fired, the matching data elements are not deleted from WM unless they are included as arguments to the <delete> system function in the RHS of the rule. The other system functions are represented in lower case and enclosed in pointed brackets (< >). Their function is described by their names. The operator - in the LHS has the same function as <not>. The symbol ! is an operator which matches the entire remaining portion of the list that contains it. Where it appears in the RHS it deposits the list that matched it but without the enclosing parentheses. The complete program is given in the following sections.

1.2.1 A Jigsaw Puzzle Production System

The representation we have chosen models the puzzle-solver as having an eye with which to focus on a single object, a hand with which to grasp an object and a limited memory which can remember a single object, a single pile, and a particular color at any one time. The puzzle is presented to the puzzle-solver as a heap (or unordered set) of puzzle pieces. The productions allow for the piling of objects, and through the functions of the eye, hand and memory, the systematic scanning of an ordered pile of puzzle pieces. Various sensing productions (those without actions in the RHS) indicate a variety of conditions of WM as is outlined below.

Working Memory Data Elements:

(PIECE p c) represents a puzzle piece where p is a unique integer associated with a piece and c is its average color. We will assume the machine knows at any time where piece p is located and so we ignore location in the representation.

(s p n c) represents the side of a puzzle piece. s is either L, R, T or B, representing the left, right, top and bottom, respectively, of piece p. Since location is ignored, rotation of a piece in space is also ignored. n is an integer representing the shape of the side where +n mates with -n and 0 represents a straight edge. c is the color

of the edge. The representation could easily be extended to represent a vector of colors for a series of points on the edge.

(NUMBER-OF-PIECES n) states that n is the total number of puzzle pieces.

(NUMBER-IN-HEAP n) states that n is the total number of pieces in the heap.

(IN-PUZZLE (PIECE p c)) states that piece p is in the puzzle.

(NUMBER-IN-PUZZLE n) states that n is the total number of pieces in the puzzle.

(JOINED ($side_1$ $side_2$) ($side_3$ $side_4$) ...) represents the sequence of sides that were joined in forming the puzzle. The symbol $side_i$ is of the form (s p n c) above.

(BEING-PUT-IN-PUZZLE x) represents the current piece x that is actively being placed in the puzzle.

(IN-HEAP (PIECE p c)) represents puzzle piece p as being in a heap with no implicit ordering of edges.

(LOOKING-AT x) states that object x is in view. If nothing is in view, then (LOOKING-AT NOTHING) is in WM.

(HOLDING x) states that x is the current object being held. If the hand is empty, (HOLDING NOTHING) is in WM.

(CURRENT-COLOR c) represents the current color being considered.

Production Memory:

LOOK-AT-PIECE-IN-HEAP:

If a piece is in the heap, then you can look at it.

```
[(LOOKING-AT =anything) $ =c1
 (IN-HEAP (PIECE =p =c) $ =object)
-(LOOKING-AT =object)
                                    -->
                                       (<delete> =c1)
                                       (LOOKING-AT =object)]
```

LOOK-AT-OBJECT-IN-HAND:

If something is in your hand, then you can look at it.

```
[(LOOKING-AT =something) $ =c1
 (HOLDING =object $ (<not> NOTHING))
-(HOLDING =something)
                                    -->
                                       (<delete> =c1)
                                       (LOOKING-AT =object)]
```

CLOSE-EYES:

If you are looking at something which is not NOTHING, then you can look away from it.

```
[(LOOKING-AT =something) $ =c1
-(LOOKING-AT NOTHING)
                                    -->
                                       (<delete> =c1)
                                       (LOOKING-AT NOTHING)]
```

OBJECT-IN-HAND-IN-VIEW:

This production senses when the object being held is in view without performing any actions.

```
[(HOLDING =object)
 (LOOKING-AT =object)
-(LOOKING-AT NOTHING)
                                    -->
                                       ]
```

PICK-UP-OBJECT-IN-VIEW:

If you are looking at an object and holding NOTHING, then you can pick up the object. Its position changes to being in your hand.

```
[(HOLDING NOTHING) $ =c1
 (LOOKING-AT =object)
 (IN-HEAP =object) $ =c3
 (NUMBER-IN-HEAP =numb) $ =c4
                                    -->
                                       (<delete> =c1 =c3 =c4)
                                       (<NUMBER-IN-HEAP (<subl> =numb))
                                       (LOOKING-AT =object)
                                       (HOLDING =object)]
```

PUT-PIECE-DOWN-IN-HEAP:

If you are holding a puzzle piece, you can put it in the heap and your hand is empty. Notice that the piece continues to be in view.

```
[(HOLDING (PIECE =p =c)) $ =c1
 (LOOKING-AT =object)
 (NUMBER-IN-HEAP =numb) $ =c3
                                  -->
                                     (<delete> =c1 =c3)
                                     (LOOKING-AT =object)
                                     (HOLDING NOTHING)
                                     (NUMBER-IN-HEAP (<add1> =numb))
                                     (IN-HEAP (PIECE =p =c))]
```

FIND-COLOR-OF-PIECE:

If you are holding a puzzle piece, you can look at it and know its color.

```
[(LOOKING-AT (PIECE =p =c))
 (CURRENT-COLOR =anything) $ =c2
                                  -->
                                     (<delete> =c2)
                                     (CURRENT-COLOR =c)]
```

FORGET-COLOR-OF-PIECE:

If a piece is in view and you know its color, you can look away from it and forget its color.

```
[(LOOKING-AT (PIECE =p =c)) $ =c1
 (CURRENT-COLOR =c) $ =c2
                                  -->
                                     (<delete> =c1 =c2)
                                     (LOOKING-AT NOTHING)
                                     (CURRENT-COLOR NOTHING)]
```

PIECE-HAS-STRAIGHT-EDGE:

This production senses when the piece in view has a straight edge.

```
[(LOOKING-AT (PIECE =p =c))
 (=any =p 0 =c)
                                  -->
                                     ]
```

PIECE-HAS-CURRENT-COLOR:

This senses when the object in view has the current color you are considering.

```
[(LOOKING-AT (PIECE =p =c))
 (CURRENT-COLOR =c)
                                  -->
                                     ]
```

EMPTY-HEAP:

This senses if the heap is empty.

```
[-(IN-HEAP =anything)
  (NUMBER-IN-HEAP 0) $ =c2
                                  -->
                                     (<delete> =c2)]
```

START-PUZZLE:

If there is no puzzle currently being built and you are holding a piece, then you can make this piece the first part of the puzzle, leaving your hand empty. Notice that you continue to look at the piece.

```
[-(IN-PUZZLE =anything)
 (HOLDING (PIECE =p =c) $ =object) $ =c2
 (LOOKING-AT =object)
                                  -->(JOINED )
                                     (<delete> =c1 =c2)
                                     (HOLDING NOTHING)
                                     (IN-PUZZLE =object)
                                     (LOOKING-AT =object)
                                     (NUMBER-IN-PUZZLE 1)]
```

PIECE-FITS-IN-PUZZLE:

This production senses when a puzzle piece that is in view will fit another puzzle piece already in the puzzle.

```
[(LOOKING-AT (PIECE =p =c) $ =object)
-(IN-PUZZLE =object)
 (IN-PUZZLE (PIECE =p2 =otherc) $ =otherp)
 (=any =p =n =anyc)
 (=another =p2 (<negative> =n) =k)
                                  -->  ]
```

FIT-PIECE-IN-PUZZLE:

If the piece being held and in view fits in the puzzle, the sides of the matching pieces can be joined.

```
[(HOLDING (PIECE =p =c) $ =object)
-(IN-PUZZLE =object)
 (LOOKING-AT =object)
 (IN-PUZZLE (PIECE =p2 =otherc) $ =otherp)
 (<forall> (=any =p =n =c) $ =c4
           (=another =p2 (<negative> =n) =k) $ =c5 )
 (JOINED ! =rest) $ =c6
                                  -->
                                     (<delete> (<all> =c4 =c5) =c6)
                                     (BEING-PUT-IN-PUZZLE =object)
                                     (LOOKING-AT =object)
                                     (JOINED ! =rest (<all> =c4 =c5))]
```

PIECE-PUT-IN-PUZZLE:

If you are holding a puzzle piece, which is currently being placed in the puzzle, you can put it down in the puzzle.

```
[(HOLDING (PIECE =p =c) $ =object) $ =c1
 (NUMBER-IN-PUZZLE =m) $ =c2
-(IN-PUZZLE =object)
 (BEING-PUT-IN-PUZZLE =object) $ =c4
                                  -->
                                     (<delete> =c1 =c2 =c4)
                                     (HOLDING NOTHING)
                                     (LOOKING-AT NOTHING)
                                     (IN-PUZZLE =object)
                                     (NUMBER-IN-PUZZLE (<add1> =m))]
```

PUZZLE-IS-FINISHED:

If all of the pieces are in the puzzle, you can stop.

```
[(NUMBER-IN-PUZZLE =n)
 (NUMBER-OF-PIECES =n)
-(IN-HEAP =anything)
                                  -->
                                     (<halt>)]
```

It should be noted that this representation is highly nondeterministic and if executed would be impossibly inefficient (even in the presence of any "syntactically-oriented" selection rules [30], as nearly 10 years of resolution theorem-proving refinement research demonstrates). It does not even "know" that it will find a solution (i.e. terminate) if it can repeat the production FIT-PIECE-IN-PUZZLE as frequently as possible, so even the crudest goal-subgoal structure is absent. (However, there is no production to remove a piece from the puzzle, which could serve as a clue to an intelligent observer.) It is even possible for the program to pick up a piece and then put it down immediately without doing anything with it.

The performance of this program could be improved to a tolerable level if the following very simple heuristic were added:

- use the following sequence repeatedly:
  - pick up a piece from the heap;
  - insert it in the puzzle if it fits;
  - if not, put it back in the heap.

An alternative heuristic would be:

- use the following sequence repeatedly:
  - pick a piece p in the puzzle with a missing neighbor;
  - pick a piece from the heap;
  - if it matches a side of p, insert it in the puzzle;
  - if not, place it back in the heap.

Each of these heuristics is very simple, consisting mainly of sequencing rules for the productions, and would seem to be within the range of automatic inference. A further improvement could be made by noting that the procedure for selecting a piece from the heap is inefficient since the same piece may be selected repeatedly (and in a production system such as OPS2 [15] which prefers to use recently referenced items in WM, in other respects a reasonable strategy, the search will almost always be ineffective). If we add a few more productions, it becomes possible to search systematically through the heap by constructing an ordered pile from the pieces in the heap, and putting a piece down in a different pile when it has looked at it. A pile is more precisely a queue, but we prefer to be consistent with puzzle terminology. The actual pile productions are given in the next section.

1.2.2 Extensions to Jigsaw Puzzle PS

Working Memory Data Elements:

(NUMBER-OF-PILES m) m is the number of piles we have made (initially zero).

(CURRENT-PILE n) represents the current pile we are using. Each pile created is assigned a unique integer, n.

(ALL-PILES p_1 p_2 ...) represents all of the piles we have made where p_i is the integer associated with a pile.

(PILE i t_1 t_2 ...) represents the contents of pile i where t_j can be any object, and t_1 is the first object in the pile.

(REMEMBERED-OBJECT t) represents object t as being tagged or remembered as special.

(REMEMBERED-PILE i) represents pile i as being special.

Production Memory:

MAKE-A-PILE:

If you want to make a pile and you are holding something, create a pile with it and your hand is empty. The object continues to be in view.

```
[(HOLDING (<not> NOTHING) $ =object) $ =c1
 (LOOKING-AT =object)
 (NUMBER-OF-PILES =m) $ =c3
 (ALL-PILES ! =r) $ =c4
 (CURRENT-PILE NONE) $ =c5
                                  -->
                                     (<delete> =c1 =c3 =c4 =c5)
                                     (NUMBER-OF-PILES (<add1> =m))
                                     (LOOKING-AT =object)
                                     (ALL-PILES (<add1> =m) ! =r)
                                     (CURRENT-PILE (<add1> =m))
                                     (PILE (<add1> =m) =object)
                                     (HOLDING NOTHING)]
```

PICK-A-PILE:

If you are not working with a pile, just pick the first one.

```
[(CURRENT-PILE NONE) $ =c1
 (ALL-PILES =p1 ! =r)
                                  -->
                                     (<delete> =c1)
                                     (ALL-PILES =p1 ! =r)
                                     (CURRENT-PILE =p1)]
```

PICK-OBJECT-FROM-PILE:

To pick an object from a pile, if your hand is empty, reach in and pick up the first one you see.

```
[(CURRENT-PILE =p1)
 (HOLDING NOTHING) $=c2
 (ALL-PILES =p1 ! =r)
 (PILE =p1 =object ! =rest) $ =c4
 (LOOKING-AT =object)
                                  -->
                                     (<delete> =c2 =c4)
                                     (LOOKING-AT =object)
                                     (HOLDING =object)
                                     (PILE =p1 ! =rest)]
```

PUT-OBJECT-IN-PILE:

To put an object in a pile, if you are holding something, just place it in the back of the pile and your hand is empty.

```
[(CURRENT-PILE =p1)
 (HOLDING (<not> NOTHING) $ =object) $ =c2
 (LOOKING-AT =object)
 (ALL-PILES =p1 ! =r)
 (PILE =p1 ! =rest) $ =c5
                                  -->
                                     (<delete> =c2 =c5)
                                     (LOOKING-AT =object)
                                     (HOLDING NOTHING)
                                     (PILE =p1 ! =rest =object)]
```

FORGET-THIS-PILE:

To forget a pile, just push the current pile behind all the others.

```
[(CURRENT-PILE =pl) $ =c1
 (ALL-PILES =pl ! =rest) $ =c2
                                  -->
                                     (<delete> =c1 =c2)
                                     (CURRENT-PILE NONE)
                                     (ALL-PILES ! =rest =pl)
```

PILE-IS-EMPTY:

A pile is empty if there is nothing in it.

```
[(CURRENT-PILE =pl)
 (ALL-PILES =pl ! =rest)
 (PILE =pl)
                                  -->
                                     ]
```

DESTROY-A-PILE:

To destroy the current pile, just strike it from memory.

```
[(CURRENT-PILE =pl) $ =c1
 (ALL-PILES =pl ! =rest) $ =c2
 (PILE =pl ! =r) $ =c3
                                  -->
                                     (<delete> =c1 =c2 =c3)
                                     (CURRENT-PILE NONE)
                                     (ALL-PILES ! =rest)]
```

THERE-ARE-NO-PILES:

There are no piles if all the piles were destroyed or none were ever made.

```
[(CURRENT-PILE NONE)
 (ALL-PILES)
                                  -->
                                     ]
```

LOOK-AT-FIRST-IN-PILE:

If there is an object at the front of the pile, you can focus on it.

```
[(LOOKING-AT =anything) $ =c1
 (CURRENT-PILE =pl)
 (PILE =pl =object $ (<not> =anything)! =rest)
                                  -->
                                     (<delete> =c1)
                                     (LOOKING-AT =object)]
```

LOOK-AT-NEXT-IN-PILE:

If you want to scan through a pile and you are looking at the first object in it, focus on the next one by placing the first object behing the last one in the pile.

```
[(LOOKING-AT =object) $ =c1
 (CURRENT-PILE =pl)
 (PILE =pl =object =next ! =r) $ =c3
                                  -->
                                     (<delete> =c1 =c3)
                                     (LOOKING-AT =next)
                                     (PILE =pl =next ! =r =object)]
```

REMEMBER-CURRENT-PILE:

Whichever pile is current, tag it as special.

```
[(CURRENT-PILE =pl $ (<not> NONE))
 (REMEMBERED-PILE =any) # =c2
                                  -->
                                     (<delete> =c2)
                                     (REMEMBERED-PILE =pl)]
```

REMEMBERED-PILE-IS-CURRENT:

This senses if the current pile is the tagged special pile.

```
[(CURRENT-PILE =pl $ (<not> NONE))
 (REMEMBERED-PILE =pl)
                                  -->
                                     ]
```

FORGET-REMEMBERED-PILE:

The special pile can always be forgotten.

```
[(REMEMBERED-PILE (<not> NONE)) $ =c1
                                  -->
                                     (<delete> =c1)
                                     (REMEMBERED-PILE NONE)]
```

REMEMBER-CURRENT-OBJECT:

As for piles, objects can also be tagged as special, when they are in view.

```
[(LOOKING-AT =object $ (<not> NOTHING))
 (REMEMBERED-OBJECT =any) $ =c2
                                  -->
                                     (<delete> =c2)
                                     (REMEMBERED-OBJECT =object)]
```

FORGET-REMEMBERED-OBJECT:

Same as for piles.

```
[(REMEMBERED-OBJECT (<not> NONE)) $ =cl
                                    -->
                                       (<delete> =cl)
                                       (REMEMBERED-OBJECT NONE)]
```

REMEMBERED-OBJECT-IN-VIEW:

This production senses when the special object is in view.

```
[(REMEMBERED-OBJECT =object)
 (LOOKING-AT =object)
                                    -->
                                       ]
```

REMEMBERED-OBJECT-IN-HAND:

This senses when the special object is being held.

```
[(REMEMBERED-OBJECT =object)
 (HOLDING =object)
                                    -->
                                       ]
```

The pile productions are very general and might be expected to be present in any system. Indeed, it seems likely that productions for dealing with heaps and piles (i.e. sets and queues) and heuristics for implementing important operations such as searching will be present in production systems as they are in modern programming languages [11].

Carrying our jigsaw puzzle problem further, sufficient productions are present now to permit simple sequencing heuristics to reflect the usual strategies used by experienced puzzle-solvers:

- work on the outside edges first (build a pile of outside edges);
- repeat the **FIT**-PIECE-IN-PUZZLE sequence until this pile is empty;
- separate the pieces in the heap into piles of the same average color and search the appropriate pile first;
- search first for pieces which have more than one neighbor of the same average color (work on the sky first);

It may appear at first glance that automatic detection of such heuristics is a task of great difficulty. Below we outline an approach to this problem which has been quite successful.

1.3 Approach

Our approach is as follows:

(1) Select a 'typical' input to the program and run the program repeatedly on this input.

(2) Record the sequence of rules selected (together with input/output information and the conflict set of rules on each cycle).

(3) Repeat this for other typical inputs.

(4) Describe the better (i.e. shorter) successful sequences in a language, CRAPS, designed for this purpose (and described in the next chapter).

(5) Generate a set of meta-rules whose objective is to aid the CRAPS description if the sequencing is inappropriate.

(6) Use the CRAPS description and the meta-rules to guide the program's subsequent decisions.

In view of the complexities of the problems with this approach, it was necessary to define a starting point from which we can proceed to study the more general case. Accordingly, we have made certain assumptions and decisions in order to derive useful results.

Inherent in our approach is the assumption that good decision-making procedures or heuristics can be inferred from the performance of the program on only selected inputs. We anticipated that the selection of inputs would be critical and that eventually new inputs would be handled incrementally, as was done by Winston [49]. However, in this

work we concentrate on the simpler problem of getting a good solution for the nonincremental case. In general, as suggested by work on learning systems [49] and information theory [6,26,40], we give preference to short CRAPS descriptions which will generate a high proportion of short successful solution sequences and few long or unsuccessful sequences.

The preliminary investigations we did suggested that the execution time of a completely declarative program will usually be too long to permit a solution except in the simpler cases. Accordingly, the solution sequences are provided by a human expert running the program in "training mode" in which its decisions are observed and corrected when necessary. Further, the human expert is aware of the structure of the program. In subsequent generalization of this approach, we anticipate the necessity of using techniques similar to those of Davis [9], which will enable the trainer to deal only with the external behavior of the program.

We also have deliberately chosen to exclude information about sequences which end in failure. It is clear that, as found by Winston and others [3,21,41,49], counterexamples will be extremely valuable. However, as the reader will note below, even the simpler problem we study poses considerable technical difficulties and it was our feeling that a clearer picture would emerge from the simpler approach.

Furthermore, the experiments described below suggest that useful results can be obtained without counterexamples.

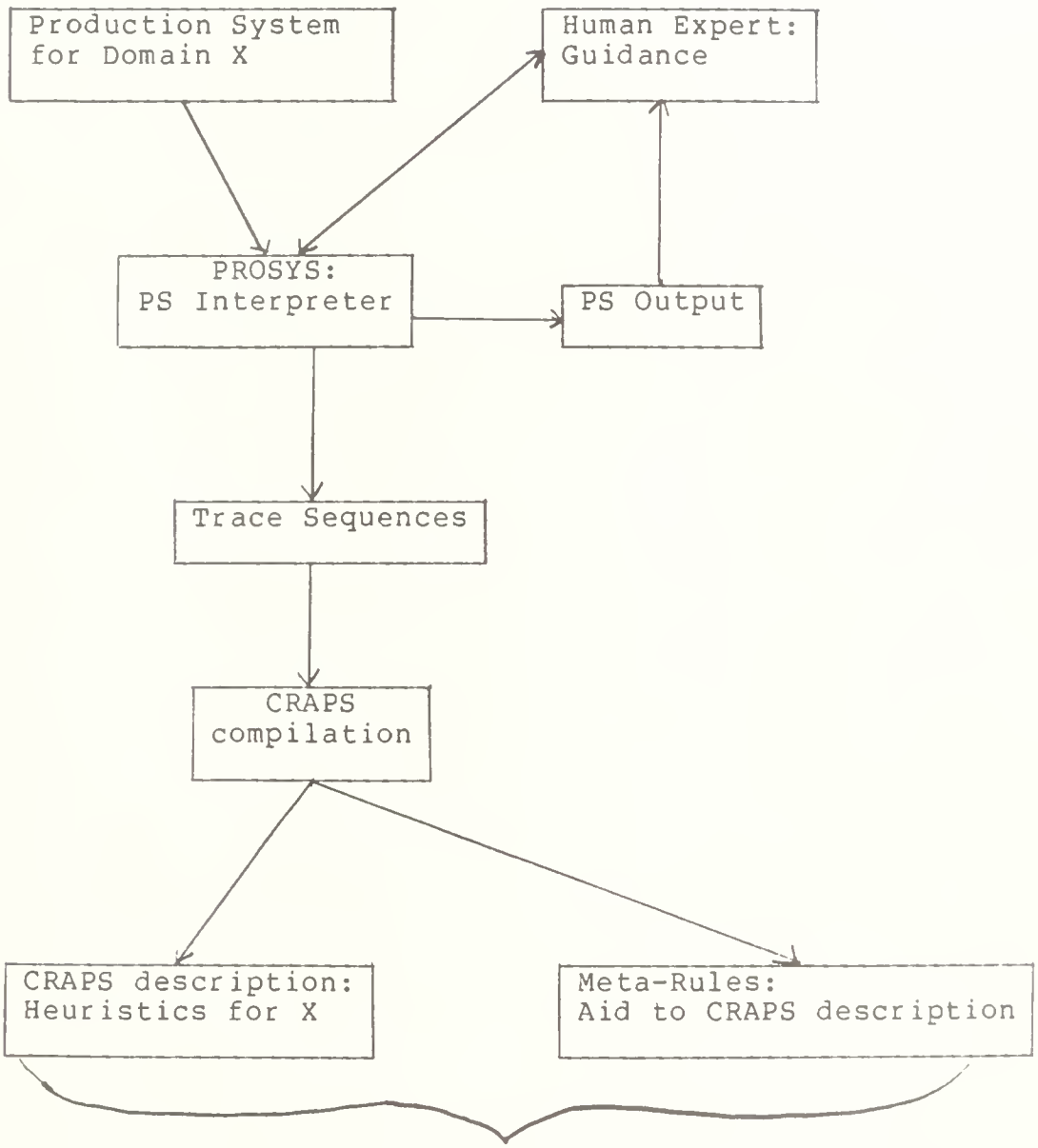
In our initial analysis, the meta-rule construct appeared to be the most ill-defined aspect of our approach. Subsequent experimentation provided us with insight as to how to define them and, consequently, how to automatically construct them. Accordingly, we decided on two directions in which to test our approach. In the first case, we attempt to infer an accurate description of several sequences which would not require the use of meta-rules. We have provided an example of this test case in Section 3.1. Alternatively, we construct an inadequate description, using a single solution sequence, which would force the use of the meta-rules. This allows us to judge the relative power and usefulness of the model we have chosen. Chapter 3 reports the findings of this experiment.

The logical organization of the overall system is outlined in Figure 1.1.

We may describe the technical problem with this approach as inductive inference of a program. The typical inductive inference problem can be formalized as follows (see for example (2)). Given a finite alphabet of symbols, Σ, and two finite subsets S, $T \subseteq \Sigma^*$ (where Σ^* denotes the set of all finite length strings composed of elements from Σ), what is the smallest positive integer k such that there is a program P (in any suitably powerful model of computation) of length equal to k (according to some well defined metric), such that if $L \subseteq \Sigma^*$ is the language accepted (or generated) by P, then $S \subseteq L$ and $T \cap L = \emptyset$? Alternatively, what is the minimum length program P, such that if $L \subseteq \Sigma^*$ is the language accepted (or generated) by P, then $S \subseteq L$ and $T \cap L = \emptyset$? Since we ignore counterexamples (T), our approach to the general problem is to attempt to construct <u>any short program P which generates all of the members of the sample set most of the time</u>. In other words, we attempt to construct a minimal length program which accurately characterizes the given sample. In formal terms, infer the shortest (nondeterministic) program P such that $S \subseteq L$, where L is the language generated by P, and when P is run randomly (making random choices at decision points in the program) for at most some large number of steps then

$$\frac{H}{G2^n}$$

Figure 1.1 System Organization.
==========



Both are fed back to the PS Interpreter for subsequent problem solution.

is maximized, where H is the number of strings generated by P that appear in S, G is the number of trials and n is the length of the program (in bits). The formula above captures the notion of the program P generating members of the sample set most of the time with the tradeoff that a program P' whose length is one more than P will be preferred if P' generates strings in S twice as often.

There are several alternative formalizations of the general problem if we consider different computational models. For instance, if P were defined to be a grammar over a finite set of symbols [23] , the problem can be viewed as inductive inference of a grammar [13]. If we restrict the power of the grammar to Type 3 [23] (S is a finite set as so is regular), the problem can be stated as the construction of the minimum length Regular Expression over Σ which generates the language. It is in fact this view of the problem which motivates the solution we present in the next chapter. As we shall see shortly, the CRAPS language in even a restricted form has the power of Regular Expressions.

1.3.1 A Complete Example

To demonstrate our approach, we will discuss a less difficult problem with an algorithmic solution: nondeterministic Binary Tree Traversal [27]. The PS program in Figure 1.2 has the ability to scan a binary tree in any order. No control information is specified, i.e., the productions encode all of the relevant information about scanning binary trees without specifying direction.

A binary tree is represented in WM by the following data format: The root of the tree is represented by (ROOT =X) where =X can be bound to any symbol. If node B has a left son A, it is represented by (LEFT B A) and similarly (RIGHT B C) represents C as the right son of B. The father B of a node A is represented by (FATHER B A). The current node being scanned is (NODE =X). Nodes can be printed only once, and so if node A has been scanned and printed, then (ALREADY-P A) is deposited in WM.

Several binary trees were placed in WM and then the program was initiated and directed by a human to perform an in-order scan of each tree. A trace for each execution was produced (see Figure 1.3). The entire set of traces was analyzed to produce the CRAPS description presented in Figure 1.4.

In this particular case the CRAPS description is a precise definition of an in-order balanced binary tree scan program. That is, given a WM specifying a balanced binary

tree, the shortest execution sequence of the program which would result in an in-order print of the nodes would belong to the set of sequences specified by the CRAPS description; furthermore, the CRAPS description provides enough information to determine which production should be fired at every point in the execution of the program. Thus a nondeterministic program has in effect been reduced to a deterministic program.

In general, we cannot assume that the CRAPS description will be as effective as this. With poor training sequences, or incorrect analyses of execution traces, CRAPS descriptions may imply heuristics which are not helpful, or which are helpful for some examples and harmful in others. We discuss these points more fully in the last chapter.

Figure 1.2 Binary Tree Scanning PS.

```
START-POINTING
[(ROOT =X) - (NODE ! =) --> (NODE =X)]

GO-LEFT
[(NODE =X) $ =Cl (LEFT =X =Y) --> (<delete> =Cl) (NODE =Y)
                                                 (FATHER =X =Y)]

GO-RIGHT
[(NODE =X) $ =Cl (RIGHT =X =Y) --> (<delete> =Cl) (NODE =Y)
                                                 (FATHER =X =Y)]

CAN-T-GO-LEFT
[(NODE =X) - (LEFT =X ! =) --> DUMMY)

CAN-T-GO-RIGHT
[(NODE =X) - (RIGHT =X ! =) --> DUMMY)

PRINT
[(NODE =X) - (ALREADY-P =X) --> (NODE =X) (<write> =X)
                                          (ALREADY-P =X)]

GO-UP
[(NODE =X) $ =Cl (FATHER =Y =X) --> (<delete> =Cl) (NODE =Y)]

STOP
[(NODE =X) $ =Cl (ROOT =X) --> (<delete> =Cl) (<halt> FIN)]
```

Figure 1.3 Execution Traces from Binary Tree PS Run.

An exact description of the contents of the trace will be deferred until chapter 2. A trace is a sequence of lists; the first item in the list is the rule applied at that point in the execution, followed by a list of the rules which were also applicable, but which were not selected for execution.

```
<<
(START-POINTING NIL (1) (2 2))
(PRINT (STOP CAN-T-GO-RIGHT CAN-T-GO-LEFT) (1 2) (3 4))
(STOP (CAN-T-GO-RIGHT CAN-T-GO-LEFT) (1 4) (5 6))
>>
<<
(START-POINTING NIL (9) (10 10))
(GO-LEFT (STOP PRINT GO-RIGHT) (9 10) (11 13))
(PRINT (GO-UP CAN-T-GO-RIGHT CAN-T-GO-LEFT) (12 13) (14 15))
(GO-UP (CAN-T-GO-RIGHT CAN-T-GO-LEFT) (12 15) (16 17))
(PRINT (STOP GO-LEFT GO-RIGHT) (9 17) (18 19))
(GO-RIGHT (STOP GO-LEFT) (9 19) (20 22))
(PRINT (GO-UP CAN-T-GO-RIGHT CAN-T-GO-LEFT) (21 22) (23 24))
(GO-UP (CAN-T-GO-RIGHT CAN-T-GO-LEFT) (21 24) (25 26))
(STOP (GO-LEFT GO-RIGHT) (9 26) (27 28))
>>
<<
(START-POINTING NIL (14) (15 15))
(GO-LEFT (STOP PRINT GO-RIGHT) (14 15) (16 18))
(GO-LEFT (GO-UP PRINT GO-RIGHT) (17 18) (19 21))
(PRINT (GO-UP CAN-T-GO-RIGHT CAN-T-GO-LEFT) (20 21) (22 23))
(GO-UP (CAN-T-GO-RIGHT CAN-T-GO-LEFT) (20 23) (24 25))
(PRINT (GO-UP GO-LEFT GO-RIGHT) (17 25) (26 27))
(GO-RIGHT (GO-UP GO-LEFT) (17 27) (28 30))
(PRINT (GO-UP CAN-T-GO-RIGHT CAN-T-GO-LEFT) (29 30) (31 32))
(GO-UP (CAN-T-GO-RIGHT CAN-T-GO-LEFT) (29 32) (33 34))
(GO-UP (GO-LEFT GO-RIGHT) (17 34) (35 36))
(PRINT (STOP GO-LEFT GO-RIGHT) (14 36) (37 38))
(GO-RIGHT (STOP GO-LEFT) (14 38) (39 41))
(GO-LEFT (GO-UP PRINT GO-RIGHT) (40 41) (42 44))
(PRINT (GO-UP CAN-T-GO-RIGHT CAN-T-GO-LEFT) (43 44) (45 46))
(GO-UP (CAN-T-GO-RIGHT CAN-T-GO-LEFT) (43 46) (47 48))
(PRINT (GO-UP GO-LEFT GO-RIGHT) (40 48) (49 50))
(GO-RIGHT (GO-UP GO-LEFT) (40 50) (51 53))
(PRINT (GO-UP CAN-T-GO-RIGHT CAN-T-GO-LEFT) (52 53) (54 55))
(GO-UP (CAN-T-GO-RIGHT CAN-T-GO-LEFT) (52 55) (56 57))
(GO-UP (GO-LEFT GO-RIGHT) (40 57) (58 59))
(STOP (GO-LEFT GO-RIGHT) (14 59) (60 61))
>>
```

Figure 1.4 Abbreviated (and 'pretty-printed') CRAPS output for Binary Tree PS example.

A formal specification of the CRAPS language is presented in Chapter 2. A CRAPS description is a sequence of lists enclosed in (double) pointed brackets (<< >>). Each list contains the name of the rule to apply or a control operation to be applied to a CRAPS (sub-)sequence. Control operations are enclosed in square brackets ([ ]).

```
<<Start-pointing
  [If (Go-left & Stop & Print & Go-right)
      Then
      <<[Repeat (While (Go-left & Go-right))
          <<[Repeat (While (Go-left & Go-right))
                    (Until (Can-t-go-left &
                               Can-t-go-right))
            <<Go-left>>]
           [Repeat (While (Can-t-go-left &
                               Can-t-go-right))
                    (Until (Go-left & Go-right))
            <<Print
              [Repeat (While (Go-up))
                      (Until (Print))
                <<Go-up>>]
              Print
              Go-right>>]>>]>>]
  Print
  [If (Go-up & Can-t-go-right & Can-t-go-left)
      Then
      <<[Repeat (While (Go-up))
                (Until (Stop))
          <<Go-up>>]>>]
   Stop>>
```

1.4 Related Research

Several similar problems have been investigated with approaches and scope overlapping with ours. We have attempted below to categorize these projects in the two areas of knowledge representation and inductive inference.

1.4.1 Knowledge Representation

Recently, many researchers have focused on the representation of knowledge in a purely declarative form with a (nearly) independent procedural component. Cohn [7], uses a control language very much like CRAPS to specify the major steps of the deductions in a formal proof system necessary to prove the correctness of a program. Sickel [39] and Deliyanni and Kowalski [10], describe an approach which automatically constructs 'links' between clauses or formulas in a formal proof system which provides guidance during the proof process. Both are essentially syntactically oriented although it can be argued that the latter is more semantically oriented.

Rychener [36] and Kibler [25] describe approaches in which the control information is more closely bound to the declarative representation, in both cases Production Systems. In effect, meta-rules are compiled into the productions

(in the form of 'control elements' in Rychener's case, and 'directions' in Kibler's case) such that a prescribed set of productions can communicate with each other (through data deposited in WM) and effect a variety of control structures. Kibler's work is the only report known to the author which specifically describes an approach to automatically generating (and compiling) meta-rules.

Davis' work [9] (specifically Chapter 7 of his thesis on strategies) laid the ground work for this thesis. The meta-rules he defined for the MYCIN system are an independent collection of rules to be applied when the next step of the deduction is not clear. The meta-rules we define in this thesis are of exactly this nature. However, his meta-rules are limited in scope (i.e. problem or data specific) and explicitly acquired (and fine-tuned) by a human expert. We present an approach to automatically generating a set of meta-rules based on the transformations of the conflict set during execution without human intervention (although the system is trained by a human from examples). To date, no research has been reported which combines automatic program generation with the production system, meta-rule methodology.

Quite similar to our overall goal is the report by Fikes, Hart and Nilsson [14]. They introduced the notion of 'robot-plans' in relation to the STRIPS problem-solving system, a formal proof system. Sequences of operator appli-

cations used in the solution of sample problems were stored for possible guidance (or planning) in solving subsequent problems. Data items that were used by and propagated through sequences of operators were stored in tables, and later used as triggers for applying the known sequences. The work we report stresses the importance of a deeper analysis of the sequence of operator applications.

1.4.2 Inductive Inference

It is very difficult to characterize much of the work in inductive inference, partly because of the scope of this rather large field. We have grouped several fields usually associated with artificial intelligence research under this topic: automatic programming, program construction, inductive inference and learning. As is often the case with such categorization, the boundary lines between these fields are often blurred or nonexistent.

The approaches to automatic program construction can be loosely termed transformational approaches. That is, starting with a high level or abstract description of what the target program should do on certain inputs, transform the specification into an executable program satisfying the specification. The specifications are in a variety of forms, including predicate calculus, set theory and lists of input/output pairs. It appears that the more difficult problems are those that have purely syntactic input (i.e. uninterpreted symbols in the specification) and only the relationship between input and output (i.e. no intermediate states). Many such systems have been developed, all with limited success. See for example Lee, Gerhart, and De Roever [28], Summers [42] and Waldinger and Lee [46]. Summers' work is particularly interesting since it is well defined and couched in a very rigorous mathematical theory of LISP programs.

The solutions to problems that have been studied which allow interpretation and intermediate states can be characterized by ad hoc, or of the heuristic search (hypothesize and test) variety. For example, Abrahams [1] reports on a system that predicts sequences of interpreted atoms based on a collection of ad hoc heuristics. Phillips' [33] work, as part of the larger PSI automatic programming project at Stanford, is based on a collection of ad hoc heuristics for constructing LISP programs.

The work we report on allows for intermediate states (the entire solution sequence) and uninterpreted symbols (only equality relates components). We provide algorithms for the analysis of sequences which we view as good approximations employing helpful heuristic rules, and are not of the more ad hoc variety. The algorithms we use do not search a large space of programs looking for a minimum solution, but instead produce one output directly (although searching of the components of the sequences is of course necessary in the analysis). We view the meta-rule construct we use as a fine-tuning mechanism, correcting the generated program whenever necessary. The optimal solution may not be required if there is an appropriate set of 'patch-ups' that allow for a more accurate solution.

Many projects have been reported which use the 'hypothesize and test' paradigm of program generation (or grammatical inference). Biermann [5] is one such system

which offers a simple but elegant heuristic search algorithm which generates Turing machines from uninterpreted intermediate states. As he points out, the model of learning which he describes has been studied by many workers in grammatical inference, for example, Feldman [13], Hedrick [21], Solomonoff [40], and Waterman [47] (to a certain extent). Hewitt's work with PLANNER [22] and the work of Schmidt and others on the Plan Recognition Problem [38] are also of this form. It seems apparent that such approaches may be effective only in the presence of very sophisticated heuristics.

Angluin's work [2,3,4] requires special mention. It is not very often that problems in artificial intelligence are defined with a high degree of rigor. She has stated the problem of Minimum Inference of Regular Expressions and proved its NP-completeness (see Garey and Johnson [17]). Given a finite alphabet of symbols Σ, two finite subsets $S,T \subseteq \Sigma^*$, and a positive integer K, is there a regular expression E over Σ that has K or fewer occurrences of symbols from Σ and such that, if $L \subseteq \Sigma^*$ is the language represented by E, then $S \subseteq L$ and $T \subseteq \Sigma^* - L$? In fact she also showed that this problem remains NP-complete if 'U' operations or Kleene star (*) operations are not allowed in the regular expressions. (This corresponds to disallowing alternation and repetition, respectively, in our analysis.) Angluin [3] elected to solve a simpler problem, namely

finding a pattern common to a set of strings where the patterns defined are different from regular expressions. Since it is believed by many researchers that NP-completeness implies intractability (see Garey and Johnson [17]), we have elected to approximate the solution of the more general problem. Notice that T as defined above corresponds to counterexamples which we do not use. As mentioned earlier, Winston's [49] work stresses the importance of counter-examples. We mention Hunt et al. [24] simply for its similarity to Winston's work.

In a more general setting, many researchers have studied inductive inference within formal syntactic systems. These approaches are based on formal inductive rules of inference. Other related work includes Meltzer [31], Plotkin [34], Popplestone [35] and Sperling [41].

Finally, in fields quite different from those mentioned here, there is some work strikingly similar to the problems we consider in this thesis. Fosdick and Osterweil [16] describe an approach to program testing based on a data flow analysis which constructs 'patterns of variable usage', which are essentially regular expressions. A classification scheme of such patterns suggests which paths in the program contain anomalies, and can then be used to predict errors in the program. Habermann [18] in quite a different application describes the use of 'path expressions', which is essentially an independent (yet integrated) collection of information used to control the synchronization of concurrent processes.

CHAPTER 2. THE CRAPS LANGUAGE

In this chapter we formally outline the specification of the CRAPS language and then present the algorithms which compile CRAPS descriptions. The UT LISP code which implements the algorithms has not been included in this thesis.

2.1 A Formal Specification

As indicated above, the CRAPS language provides a semantic framework with which to specify or describe sequences of rule applications in the execution of the nondeterministic program. The basic primitive of CRAPS is called a *unit*. A unit specifies either a rule application with preconditions, in which case it is called a *simple* unit, or a control operation applied to a sequence of units. The control operations, which are formally defined in the next section, are (implicit) *C*oncatenation of units producing sequences, *R*epetition of a sequence controlled by simple Boolean assertions in Disjunctive Normal Form (DNF), *A*lternative or conditional selection of a sequence from a set of sequences and the *P*ermutation of a set of sequences. (From which we have derived the acronym CRAPS.)

The CRAPS operators correspond to various control primitives of conventional programming languages. Concatena-

tion corresponds to sequential execution of statements and the repetition operator corresponds to iteration statements with continuation and termination conditions. The alternation operator specifies alternative sequences of actions much like an ALGOL-68 'case' statement or LISP conditional expression. The permutation primitive represents a form of concurrent execution similar to the ALGOL-68 'collateral expression'.

2.1.1 A BNF Description of CRAPS

In what follows, parentheses are terminal symbols. Alternatives in the right-hand side of a production are separated by lines and preceded with ::= . The nonterminal <atom> is any (LISP) identifier.

```
 <sequence>      ::=<< <unit+> >>
 <sequence+>     ::=  <sequence> <sequence+>
                 ::=  <sequence>
 <unit+>         ::=  <unit> <unit+>
                 ::=  <unit>
 <unit>          ::=  <simple-u>
                 ::=  <repetition-u>
                 ::=  <permutation-u>
                 ::=  <atom-u>
                 ::=  <alternation-u>
 <simple-u>      ::=   (<atom> <dnf>)
 <atom*>         ::=  <atom> <atom*>
                 ::=
 <repetition-u>::=  (R.* <sequence>
                       (W.* <dnf>)
                       (U.* <dnf>) )
 <dnf>           ::=  ( OR.* ( <atom*>) <disjuncts+>)
                 ::=  ( <atom*> )
 <disjuncts+>    ::=  ( <atom*> ) <disjuncts+>
                 ::=  ( <atom*> )
 <permutation-u> ::= (P.* <sequence><sequence+>)
 <atom-u>        ::=  <atom>
<alternation-u>::=  (A.* <cond> <cond*>)
 <cond>          ::=  (<sequence> <dnf>)
 <cond*>         ::=  <cond> <cond*>
                 ::=
```

The syntax specified is used in the actual implementation. However, a 'pretty-printed' version of the language is used in the machine output contained in this thesis. Consequently, R.* is replaced by REPEAT, P.* by PERMUTE, and A.* by an IF THEN ELSE form, each enclosed in square brackets (see Appendix 2 and Figure 1.4).

2.1.2 Definition of the CRAPS Operators

Cambridge notation is used for the operators except for concatenation which is implicitly specified as a sequence of units.

1. Concatenation of units.

Let $s_1, s_2, \ldots, s_n$ be units. Then the sequence $S = \langle\langle s_1\ s_2\ \ldots\ s_n \rangle\rangle$ is the concatenation of the units s_i, $i = 1,\ldots,n$. (Throughout the remainder of this paper, sequences will be represented by a sequence of objects enclosed in pointed brackets, << and >>, and separated by blanks.

2. Concatenation of sequences.

Let $S_1 = \langle\langle s_{11}\ s_{12}\ \cdots\ s_{1m_1} \rangle\rangle$

$S_2 = \langle\langle s_{21}\ s_{22}\ \cdots\ s_{2m_2} \rangle\rangle$

$\vdots$

$S_j = \langle\langle s_{j1}\ s_{j2}\ \cdots\ s_{jm_j} \rangle\rangle$

Then $\langle\langle S_1\ S_2\ \ldots\ S_j \rangle\rangle =$

$\langle\langle s_{11}\ s_{12}\ \cdots\ s_{1m_1}\ s_{21}\ s_{22}\ \cdots\ s_{2m_2}\ \cdots\ s_{j1}\ s_{j2}\ \cdots\ s_{jm_j} \rangle\rangle$

In the obvious way, we denote

$$S^n = \begin{cases} \langle\langle S^{n-1}\ S \rangle\rangle & \text{if } n \geq 1 \\ \text{the null sequence} & \text{if } n = 0 \end{cases}$$

3. Repetition of a sequence.

Let S_1 be as in 2 above. Then

$(R.*\ S_1\ (W.*\ c_1)\ (U.*\ c_2)) = S_1^n$

where $n \geq 1$ is chosen so that repetition terminates as soon as c_1 is false or c_2 is true. The conditions c_1 and c_2 are simple Boolean assertions of rule applicability written in DNF (see 6 below).

4. Alternation of a set of sequences.

Let $S_1, S_2, \ldots, S_j$ be as in 2 above.
Let $U = (A.*\ (S_1\ c_1)\ (S_2\ c_2)\ \ldots\ (S_j\ c_j))$, $j \geq 1$, and c_i, $i = 1, \ldots, j$ are simple Boolean assertions in DNF. Then

$$U = \begin{cases} \text{if } c_1 \text{ is true then } S_1 \\ \quad \text{else if } c_2 \text{ is true then } S_2 \\ \quad\quad \text{else } \ldots \\ \quad\quad\quad \text{else if } c_j \text{ is true then } S_j \\ \quad\quad\quad\quad \text{else } S^0. \end{cases}$$

5. Permutation of two or more sequences.

Let $S_1, S_2, \ldots, S_j$ be as in 2 above. Then

$(P.*\ S_1\ S_2\ \ldots\ S_j) =$

$$\left\{ \langle\langle s_{i_1 k_1}\ s_{i_2 k_2}\ \ldots\ s_{i_\ell k_\ell} \rangle\rangle \mid \ell = \sum_{t=1}^{j} m_t \text{ , and if } i_r = i_q \text{ then } k_r < k_q \right\}.$$

The permutation operator is more like a shuffle operator, that is the argument sequences can be merged in any order as long as the ordering within each sequence is maintained.

6. DNF conditions.

Let C = (OR.* (a_{11} a_{12} ... a_{1n_1})
⋮
(a_{k1} a_{k2} ... a_{kn_k}))

Then C is true if $1 \leq \exists i \leq k$ such that $1 \leq \forall j \leq n_i$, a_{ij} is active, otherwise C is false. If C = (a_1 a_2... a_k), then C is true if $1 \leq \forall j \leq k$, a_j is active.

A simple unit is the most basic primitive of CRAPS. It is composed of two parts, the name of a rule to apply and a expression that must be sa' ᶜied (i.e. evaluate to true) in or to execute the associated rule name.

The (programmed) interpretation of the above definitions follows: A sequence of units specifies sequential application of each of the component units. The repetition of a sequence is much like a PL/I do loop; apply the argument sequence one or more times while the condition c_1 is true and c_2 is false.

The alternation operator selects the first sequence to apply whose corresponding condition, c_i, is true. The permutation of a set of sequences specifies that each of its argument sequences are to be applied in any order or merged in any order; sequential execution of the appended arguments is chosen.

Finally, in the execution of a production system, whenever a DNF condition is to be evaluated, the condition

is true if any of its conjuncts are true; otherwise it is false. A conjunct is true if each of the productions listed are matched by data from working memory at the time the condition is being evaluated. Examples of these definitions follow.

2.1.3 Examples

(1) When executing the simple unit

(GO-LEFT (OR.* (STOP PRINT) (GO-RIGHT))),

if the productions STOP, GO-RIGHT and GO-LEFT are active, then the rule named GO-LEFT would be executed next.

(2) (R.* <<(GO-RIGHT(PRINT))

(PRINT (OR.* (GO-RIGHT) (STOP)))>>

(W.* (GO-RIGHT))

(U.* (STOP)))

produces the sequencing:

<<(GO-RIGHT (PRINT))

(PRINT (OR.* (GO-RIGHT) (STOP)))

(GO-RIGHT (PRINT))

(PRINT (OR.* (GO-RIGHT) (STOP)))

⋮

>>

continually while the production GO-RIGHT is active and STOP is inactive. The conditions are tested when the end of the argument sequence is reached. Notice,

however, that the preconditions of the units contained in the argument sequence must still be satisfied when executing the argument sequence. The details are in Section 2.1.4.

(3) (P.* <<(START-ENGINE( )) (SHIFT-FORWARD ( ))>>
<<(TURN-ON-RADIO ( )) >>)

could produce 3 sequences:

{<<(START-ENGINE())(SHIFT-FORWARD())(TURN-ON-RADIO())>>
<<(START-ENGINE())(TURN-ON-RADIO())(SHIFT-FORWARD())>>
<<(TURN-ON-RADIO())(START-ENGINE())(SHIFT-FORWARD())>>
}

In a parallel environment (two arms for a robot) both sequences could be done simultaneously. In the actual implementation of the language, the argument sequences are concatenated and executed in order.

(4) (A.* (<<(**EAT** ( ))>> (THERE-IS-FOOD))
(<<(THANK-HOST ( ))>> (PLATE-EMPTY)))

produces the sequencing <<(**EAT** ( ))>> if the production THERE-IS-FOOD is active, otherwise, <<(THANK-HOST ( ))>> if the production PLATE-EMPTY is active, or it produces no sequencing at all.

2.1.4 Implementation

The implementation of the operators is very straightforward. The PS interpreter maintains a stack of units corresponding to the CRAPS description that is in control of the sequencing. Successive units are popped from the stack and applied. If the stack is empty, or the precondition of a simple unit is false, the meta-rules are called. The algorithm of figure 2.1 is invoked whenever a rule is to be selected from the conflict set. The details of the meta-rule implementation can be found in the next section.

Figure 2.1 Implementation of CRAPS.

```
repeat forever; Apply the rule selected as follows:

   case;

     (Stack Empty):
       Set P to meta-rule choice;
       If P not empty then select P from conflict set;
       else stop;  end if;

     (Simple unit on stack):
       If the DNF of the unit is true and the rule name is active
          then
            pop the stack;
            select the rule name from the conflict set;
          else
            set P to the meta-rule choice;
            if P is not empty then select P from conflict set;
              else pop the stack; end if;
          end if;

     (Permutation unit on stack):
       pop the stack;
       push the appended argument sequences on stack;

     (Repetition unit on stack):
       if the While component, and the Until component are both
          empty then
          pop the stack;
          push argument sequence on stack; /* executed once */
         else
          if While condition is true and Until is false then
            push argument sequence on stack;
           else
            pop the stack;
          end if;
        end if;

     (Alternation unit on stack):
       Scan each argument of unit;
         if the DNF condition is true then
           pop the stack;
           push the corresponding argument sequence;
         end if;
       end scan;
       if none were true then pop the stack;
  esac;

end repeat;
```

2.2 Meta-Rules

There are four types of meta-rules which assist a CRAPS description in controlling a PS. It is the simple unit which actually selects the next rule to fire on each cycle (the higher level control units produce sequences of simple units), and if in the event that the DNF expression evaluates to false or the specified rule is not active, the meta-rules are called upon to suggest a list of rule names to try. For example, suppose that the simple unit (A (B C)) is in control, E was the previously fired production, and the current conflict set of rules is {B D}. This situation may be described as:

(1) A and C should be active

(2) D should (perhaps) be inactive

(3) {B D} is currently active

(4) E was just fired.

Accordingly, the meta-rules which have been implemented are designed to deal with the four cases listed. In each case, a meta-rule may suggest a list of rules to try in that situation. The suggestions are weighted since a rule may be suggested several times by different meta-rules. Each rule is scanned and its LHS tested; if it evaluates to true, the corresponding RHS is executed. The exact definitions of the meta-rules appear in the next section.

2.2.1 A BNF Description of the Meta-Rules

The same conventions are followed as in 2.1.1.

```
<meta-rule>    ::=  [<m-LHS> --> <m-RHS>]
<m-LHS>        ::=  (Want-active (<atom*>))
               ::=  (Want-inactive (<atom*>))
               ::= (Currently-active (<atom*>))
               ::=  (Just-fired (<atom*>))
<m-RHS>        ::=  (Try-to-fire (<atom*>))
<atom*>        ::= <atom> <atom*>
               ::= <atom>
```

The following is an example of a meta-rule:

```
[(Want-active(A  C)) -->
                    (Try-to-fire(G))]
```

Others can be found in Figure 3.3.

2.2.2 Definition of the Primitives

Let P' be a list of rule names

J be the rule fired on the previous cycle

D be the union of all the names appearing in the DNF of the current simple unit in control

U be the rule specified by the current simple unit

C be the current set of active rules.

The primitive function Want-active is a Boolean function which tests the current state of the CRAPS description and the currently active set of rules. It is defined as follows:

$$(\text{Want-active } P') = \begin{cases} \text{true if } P' \subseteq D \cup \{U\} \\ \text{else false} \end{cases}$$

Similarly, the other primitive functions are defined as follows:

$$(\text{Want-inactive } P') = \begin{cases} \text{true if } P' \subseteq C - (D \cup \{U\}) \\ \text{else false} \end{cases}$$

$$(\text{Currently-active } P') = \begin{cases} \text{true if } P' \subseteq C \\ \text{else false} \end{cases}$$

$$(\text{Just-fired } P') = \begin{cases} \text{true if } J \subseteq P' \\ \text{else false} \end{cases}$$

The definitions are very simple and so the details of the implementation are not included.

The function Try-to-fire deposits a subset of its argument list of rule names in a master list (TRYLIST) maintained by the interpreter. All suggested rules must be active. The most often suggested rule is selected for execution; in the case of ties, preference is given to the rule name in the current simple unit, U.

Try-to-fire is defined as:

```
Try-to-fire(P'):procedure;
  TRYLIST: =
    TRYLIST ∪ (P' ∩ C); /* but duplicates remain */
  end;
```

2.3 Analyzing Sequences

When running a production system to a successful conclusion, a trace of the actions performed by the PS is generated. A series of these traces, called input trace sequences, are presented to a system which analyzes and produces CRAPS descriptions from them. The input sequences, which are described in the next section, are transformed to a variety of intermediate states until a final CRAPS sequence is constructed.

2.3.1 Input Trace Sequences

The exact form of an input sequence is $<<P_1\ P_2\ \ldots\ >>$ where P_i , called a *trace unit*, is of the form:

$$(P_i'\ (P_{i1}\ P_{i2}\ \cdots\ P_{it_i})\ (N_{i1}\ N_{i2}\ \cdots\ N_{ij_i})\ (M_{i1}\ M_{i2}))\ .$$

P_i' is the name of the rule which was applied on the i^{th} cycle of the execution of the PS during the training session. $(P_{i1}\ P_{i2}\ \cdots\ P_{it_i})$ is the set of rules which matched the data environment during the i^{th} cycle. This set encapsulates the data environment at that point in the execution, and is used to calculate the DNF conditions for the units and the meta-rules.

The third component of the trace unit, $(N_{i1}\ N_{i2}\ \cdots\ N_{ij_i})$, is the set of unique integers associated with the instances of data elements which matched the pattern of the rule P_i'. Finally, $(M_{i1}\ M_{i2})$ is the range of unique integers associated

with all of the data elements produced by the action portion of the rule P_i'. The last two pieces of information allow for the construction of a data flow graph.

The trace sequence is an exact history of the execution of the nondeterministic program and is therefore already partially ordered by the 'back-dominance relation.' Every trace unit within the sequence is preceded by those trace units which produced data for it. This partial ordering simplifies much of the subsequent analysis of the trace sequence.

Figure 1.2 is an example of a trace sequence produced by a run of the binary tree scanning PS.

2.3.2 Notation and Terminology

The algorithms which follow are presented in a SETL-like language which makes use of n-tuple and set notation. Lower case symbols in the code represent key words with obvious meanings. All sequences will be represented by vectors whose enclosing brackets are < and > . Each item of a sequence is referenced by integer indices. The null sequence is represented by the symbol omega. Calls to subroutines will be denoted by prefix notation with arguments enclosed in parentheses. The binary operators used (for example +) have various interpretations depending upon the type of data structure supplied to them. For example, + can represent set union, vector concatenation or integer addition.

The input trace units are transformed to a variety of intermediate forms until a final sequence of CRAPS units is produced. A trace unit will be called a T-unit; a sequence of trace units, a T-SEQUENCE; a sequence of units in intermediate form an I-SEQUENCE; and a CRAPS sequence a C-SEQUENCE. The intermediate, simple, atom, permutation, repetition and alternation units will be termed, respectively, I-, S-, A-, P-, R-, and ALT-units. Each type of unit will be represented by a record type data structure with named components.

Declarations:

```
T-unit: ( PNAME: atom,
          TRUE-PS: set of atoms,
          IN-LIST: set of integer,
          OUT-LIST: [1:2] integer,
          SONS: [1:] integer);

I-unit: ( NUMBER-PNAME:(NODENUM: integer,
                        PNAME: atom),
          TRUE-PS: as in T-unit,
          IN-LIST: as in T-unit,
          OUT-LIST: as in T-unit,
          SONS: as in T-unit);

S-unit: ( NUMBER-PNAME: as in I-unit,
          TRUE-PS: as in T-unit);

A-unit: ( NAME: atom,
          ORDERING: integer,
          EQUIVALENTTO: A-unit,
          SUBSEQNC: P-unit);
```

Each P-, R- and ALT-unit begins with the signifier P.*, R.* and A.*, respectively. The remaining components of each are:

```
P-unit: ( SUBSEQUENCES: [1:(>=2)] C-SEQUENCE);

R-unit: ( LENTH: integer,
          C-SEQ: C-SEQUENCE,

          WHILE: (atom= 'W.*',
                  DNF: Boolean expression),

          UNTIL: (atom = 'U.*',
                  DNF: Boolean expression) );

ALT-unit: ( [1:(>=2)] COND: ( C-SEQ: C-SEQUENCE,
                              DNF: Boolean expression));
```

In the discussion of graphs, ordinary graph notation will be used, Nodes, edges and paths will be referenced by subscripted v's, e's, and p's, respectively.

2.3.3 Algorithms

2.3.3.1 Top level control

The top level algorithm repeatedly reads in T-SEQUENCES and processes each one by calling a succession of routines. It begins by constructing a graph which identifies all of the data dependencies within a sequence. This graph is then linearized by the permutation routine to obtain a preliminary description of the sequence. The repetition algorithm is then called to produce descriptions which possibly contain R-units. All sequences read in are processed in this fashion, producing a set of descriptions which are input to the alternation detection algorithm. The final output is a CRAPS description. See Figure 2.2 for complete details of this algorithm. An example of an actual run of the system is exhibited in Appendix 2.

Note that constructing units in this order prohibits the appearance of alternation within a repetition and permutation. Because of the difficulty of the pattern analysis considered here, less powerful CRAPS descriptions are generated than can be specified in the language. A discussion of these points is deferred until Chapter 3.

Figure 2.2: Top level control.

```
DESCRIBE(): procedure;

NODE-COUNT: integer; /*assignment of unique node numbers*/
PCOUNT: integer; /*assignment of artificial ordering*/
PROD-NAMES: set of atoms; /*production names*/
PSTARNAMES: set of A-units;
VISITED: [1:] I-SEQUENCE; /*indexed by node numbers and used
                  for linear sequence constructed from DAG*/
NODEPTRS: [1:] I-unit; /*indexed by node numbers*/
ND: [1:] integer; /* numbers assigned to nodes of
                    DAG for Tarjan's algorithm*/
DFSN: [1:] integer; /*depth first spanning tree numbers for
                      the algorithm*/
DFSNUMBER: integer; /*used for values of DFSN*/
SEQ: T-SEQUENCE or I-SEQUENCE or C-SEQUENCE or atom;
DESCS: set of I-SEQUENCE;
ENVIRONS: set of pairs of sets; /*for meta-rule construction*/
MADE-ACTIVE: like ENVIRONS;
MADE-INACTIVE: like ENVIRONS;
JUST-FIREDS: like ENVIRONS;

/* Initialization */

     PCOUNT := 0;
     DFSNUMBER := 0;
     NODE-COUNT := 0;
     PSTARNAMES := {};
     PROD-NAMES := {};
     DESCS := {};
     ENVIRONS := {};
     MADE-ACTIVE := {};
     MADE-INACTIVE := {};
     JUST-FIREDS := {};

/* MAIN INPUT LOOP */

     repeat while( SEQ \= omega ) doing( read(SEQ) );

/* We assume production names have an ORDERING component. */

        if SEQ is a production name then
             SEQ.ORDERING := PCOUNT;
             PCOUNT := PCOUNT + 1;

        else

/* Preparation for processing an input trace sequence. */

             LENTH := length(SEQ);
             VISITED[1:LENTH] := omega;
             NODEPTRS[1:LENTH] := omega;
             DFSN[1:LENTH] := 0;
             ND[1:LENTH] := 0;

/* Assign unique node numbers to each T-unit */

             SEQ := CHANGEP-FIRED(SEQ);

/* Construct the DAG and Meta-rules. */
```

```
                SEQ := INTERVAL-OF-EXIT(SEQ);

/* Construct permutation units */

                SEQ := CONSTRUCT-P*(SEQ);

/* Repetition detection. Notice () is sent as the
 second argument since SEQ is not followed by any
 unit.  See REPEATS for a description of this.*/

                SEQ := REPEATS(SEQ,());
                NODECOUNT := LENTH + NODECOUNT;
                DESCS := DESCS +  {SEQ};

            end if SEQ;

        end repeat;

/* FINAL PROCESS */

/* We now collapse all of the sequences into one CRAPS
   description. */

        SEQ := ALTERNATIVES(DESCS);

/* All of the atom units are replaced by their corresponding
  permutations.  The following routine is a simple scanning
  procedure which does the replacement. It is not shown. */

        SEQ := REPLACE-ATOMS(SEQ);

/* We can now remove the numbering of simple units and the
   lengths within the repetitions.  The numbering was
   necessary for duplicates within the permutations. All such
   duplicates can now be removed also.  The following is a
   simple scanning routine which does the removal and is not
   shown.  */

        SEQ := REMOVENUMSANDDUPS(SEQ);

/* The description and meta-rules have been constructed so... */

        print(SEQ,ENVIRONS,MADE-ACTIVE,MADE-INACTIVE,JUST-FIREDS);

end DESCRIBE;
```

2.3.3.2 DAG Routines

The input sequence is scanned and a unique number is assigned to each instance of a rule application in the T-units. A Directed Acyclic Graph (DAG) is then constructed with nodes (indexed by the unique numbers) corresponding to rule applications and edges representing the flow of data from one rule to another. This DAG is constructed in an obvious manner from the IN-LIST and OUT-LIST components of the T-units. It is assumed that the last unit in the sequence signalled success of the problem solution and therefore its corresponding node is interpreted as the unique sink node of the DAG (see page 63 for examples).

The DAG completely specifies all of the data dependencies between rules and therefore all of the possible execution sequences. In general, an arc from node v_i to node v_j indicates a concatenation of the subsequence associated with v_i (leading to and including v_i) with the unit associated with node v_j.

The DAG is first cleansed of irrelevant arcs, called forward arcs. An arc e, from node v_i to node v_j is a forward arc if there is some directed path p from v_i to v_j which does not include e as an intermediate arc. A forward arc specifies that node v_i must precede v_j and that node v_i precedes the nodes on path p. Since the nodes on path p enter v_j, they too must precede v_j. Clearly, the forward arc is redundant and can be removed from the DAG, which prohibits

the construction of irrelevant permutations.

The forward arc removal algorithm is due to R. Tarjan [43,44,45]. It proceeds as follows:

Let DFSN(i) be a Depth-first Spanning Tree numbering assigned to node i beginning the numbering from the sink node of the graph. Let ND(i) be the sum of the numbers assigned to the immediate predecessors of node i. Then there is a path p from v_i to v_j iff DFSN(i) $\leq$ DFSN(j) $\leq$ DFSN(i)+ND(i). Given a node v and the set of entering edges, it is easy to check from this inequality which edges are forward arcs. For any two immediate predecessors, v_i and v_j , if the above inequality holds, then the edge from v_j to v can be deleted. Figure 2.3 contains complete details of the algorithm.

Figure 2.3: DAG construction and cleansing.

```
CHANGEP-FIRED(SEQ): procedure;

SEQ: T-SEQUENCE;

     CNTR := 0;

     1 <= forall i <= #SEQ;

/* The PNAME component is replaced by a NUMBER-PNAME
   component producing I-units. */

          SEQ[i].PNAME := <CNTR, SEQ[i].PNAME>;
          CNTR := CNTR + 1;

     end forall i;

end CHANGEP-FIRED;


INTERVAL-OF-EXIT(SEQ): procedure;

SEQ: I-SEQUENCE;

/* The last unit of SEQ is assumed to be the unique sink
   of the DAG to be constructed by this routine.  SEQ is
   partially ordered by the back-dominance relation. */

     #SEQ >= forall i >= 1;
          NODEPTRS[SEQ[i].NODENUM] := SEQ[i];

          #SEQ >= forall j > i;
               if exists n in SEQ[i].IN-LIST such that
                   SEQ[j].OUT-LIST[1]
                                  <= n <= SEQ[j].OUT-LIST[2]
                   then
                   SEQ[i].SONS := SEQ[i].SONS
                                          + <SEQ[j].NODENUM>;
               end if;

          end forall j;

/* Meta-rule construction is done at this point, see figure
   2.7.  The definitions in section 2.2 suggest simple set
   expressions between the data environments(conflict sets)
   between successive input units. */

          CONSTRUCT-M-RULE(SEQ[i-1],SEQ[i]);

     end forall i;

     return CLEAN(SEQ);

end INTERVAL-OF-EXIT;


CLEAN(SEQ): procedure;

SEQ: I-SEQUENCE;

/* IN-LIST and OUT-LIST components are no longer useful. */
```

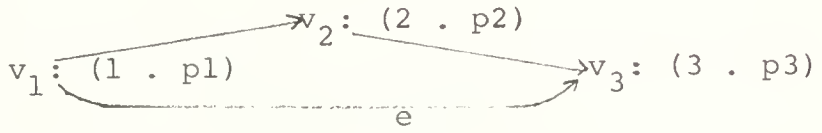
```
      1 <= forall i <= #SEQ;
           SEQ[i].IN-LIST := omega;
           SEQ[i].OUT-LIST := omega;
      end forall i;

/* We are now ready to process the DAG after we have reversed
  SEQ so that the sink node is first. */

      SEQ := reverse(SEQ);


/* Now we use Tarjan's algorithm to remove forward arcs.*/

      REMOVEFWDARCS(SEQ);
      SEQ := reverse(SEQ);
      return SEQ;

end CLEAN;


REMOVEFWDARCS(SEQ): procedure;

SEQ: I-SEQUENCE;

/* Tarjan's algorithm. */

      DFSNUMBER := 1;
      DEPTH-FIRST-SEARCH(SEQ[1]);

      1 <= forall i <= #SEQ;

           if #SEQ[i].SONS >= 2 then

              forall child1 in SEQ[i].SONS;

                 forall child2 in SEQ[i].SONS - {child1};

                    if TARJAN-INEQUALITY(child1,child2) then

                       /* remove the arc , child1 = omega */
                       SEQ[i].SONS := SEQ[i].SONS - {child1};

                    end if;

                 end forall child2;

              end forall child1;

           end if;

       end forall i;

end REMOVEFWDARCS;

TARJAN-INEQUALITY(N1,N2): procedure;

N1,N2: integer; /* node numbers */

      if DFSN[N1] <= DFSN[N2] <= DFSN[N1] + ND[N1]
       then
         return true
      else return false;
      end if;

end TARJAN-INEQUALITY;
```

```
DEPTH-FIRST-SEARCH(UNIT): procedure;

UNIT: I-UNIT;

      if DFSN[UNIT.NODENUM] = 0
       then
           DFSN[UNIT.NODENUM] := DFSNUMBER;
           DFSNUMBER := DFSNUMBER + 1;
           V, ND[UNIT.NODENUM] := 1 +
                        +:{DEPTH-FIRST-SEARCH(NODEPTRS[s]) !
                                    forall s in UNIT.SONS};
           return v;
       else return 0
       end if;

end DEPTH-FIRST-SEARCH;
```

2.3.3.3 Permutation Routines

The construction of permutations is based on the observation that if a node has two or more entering edges then the subsequences associated with its predecessors can be permuted. The partial ordering makes this construction straightforward. Using a simple sequential scan, it is assured that the sequences associated with its predecessors have already been constructed when a node is to be processed.

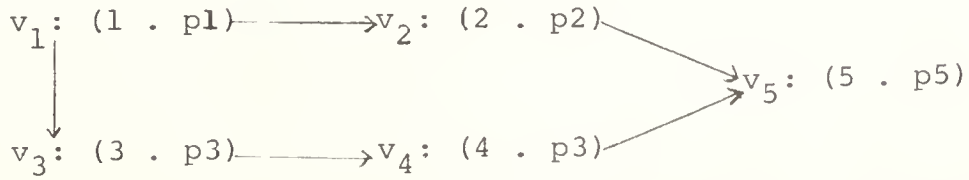
Forward arc removal prevents an irrelevant permutation from being constructed. Consider the following DAG (we have removed the TRUE-PS field for simplicity):



When node v_3 is to be processed, the following subsequence would be constructed: $\langle\langle(P.^* \langle\langle v_1\ v_2\rangle\rangle \langle\langle v_1\rangle\rangle)\ v_3\rangle\rangle$. The permutation is obviously irrelevant and if arc e were ignored then the subsequence constructed would be $\langle\langle v_1\ v_2\ v_3\rangle\rangle$.

The permutations are further processed by first factoring out common leading subsequences and then applying the repetition detection procedure so that equivalences with other permutations could be more easily recognized.

Consider the following graph:



The subsequences associated with nodes v_1 , v_2 , v_3 and v_4 are, respectively, <<v_1>>, <<v_1 v_2>>, <<v_1 v_3>>, <<v_1 v_3 v_4>>. When processing v_5 the following would be constructed: <<(P.* <<v_1 v_2>> <<v_1 v_3 v_4>>) v_5>>. Each of the subsequences begins with node v_1. By definition of permutation, this is equivalent to <<v_1 (P.* <<v_2>> <<v_3 v_4>>) v_5>>. This final form is less complex and more closely represents the control information in the DAG. Notice that if v_3 were instead (6 . P2) we could not factor out nodes v_2 and v_3 since they are distinct instances of applying rule p2 and could not be identified as the same. The final permutation can be further condensed by replacing the subsequence <<v_3 v_4>> by an R-unit specifying a repetition of p3.

The permutation units are then replaced by unique A-units after being normalized by a lexicographical ordering of the argument subsequences. The new A-units are then collected in a list and compared with previous A-units for equivalence. Normalizing makes this search considerably easier; replacing all equivalent A-units by a single one makes the repetition detection procedure considerably faster. When looking for repeating subsequences, the only units that can be compared to a permutation is another such permutation. For example, given <<(P.* <<B>><<A>>)A>> where A and B are arbitrary units, the inner subsequence <<A>> cannot be viewed as a repetition with the outer unit A because of the presence of B within the P-unit. The B must precede

the outer A and in particular can occur between the two occurrences of A. See Figure 2.4 for complete details of these algorithms.

Figure 2.4. Permutation Construction.

```
CONSTRUCT-P*(SEQ): procedure;

SEQ: I-SEQUENCE;

/* CURRENT accumulates the linear sequence. */

     CURRENT := omega;

     1 <= forall i <= #SEQ;
          NN := SEQ[i].NODENUM;

/* The following assures us that unique node numbers
   will be assigned to all production names in every
   instance of a trace unit in which it occurs. */

          SEQ[i].NODENUM := NODECOUNT + NN;

/* If this node has no sons, it was preceded by nothing
  which produced input for it in the sequence and it
  is appended to no other sequence constructed.
  Notice that when a node is placed in the linear
  sequence, the SONS field (arcs) are removed. */

          if SEQ[i].SONS = omega then
               SEQ[i].SONS := omega;
               CURRENT := <SEQ[i]>;

           /* If it has only one son, then a simple
              concatenation is performed. */

          else if #SEQ[i].SONS = 1 then
                  CURRENT := VISITED[s in SEQ[i].SONS];
                  SEQ[i].SONS := omega;
                  CURRENT := CURRENT + <SEQ[i]>;
               else

                  /* a P-unit is to be constructed. */

                  CURRENT := +:{ {VISITED[s]}  !
                           forall s in SEQ[i].SONS };

                  /* factor out common leading subsequences. */

                  FACTOR(CURRENT,PREFIX);
                  SEQ[i].SONS := omega;

                   /* CURRENT will become the
                      SUBSEQUENCES component of the newly
                      formed P-unit after repetition detection
                      is applied. */

                  CURRENT := +:< <REPEATS(s,SEQ[i])> !
                                 forall s in  CURRENT >;
                  NEWUNIT := heap(P-unit);
                  NEWUNIT.SUBSEQUENCES := CURRENT;
                  CURRENT := PREFIX +
                                 <PSTARS(NEWUNIT)>
                                   +
                                  <SEQ[i]>;
               end if #SEQ[i].SONS;
```

```
          end if SEQ[I].SONS = omega;

          VISITED[NN] := CURRENT;

     end forall i:

/* The final value of CURRENT is the subsequence
  associated with the last unit of the sequence which
  is assumed to be the unique sink node of the DAG.
  Notice how extraneous nodes in the graph will auto-
  matically be lost since they would not be part of the
  sequence associated with the sink.  A node is extraneous
  if it does not contribute to the solution sequence, i.e.,
  it does not lie on any path terminating at the sink node.*/

     return CURRENT;

end CONSTRUCT-P*;


FACTOR(CURRENT,PREFIX): procedure;

CURRENT: set of I-SEQUENCE;
PREFIX: I-SEQUENCE;

     PREFIX := omega;
     TARGET := any one in CURRENT;
     CURRENT := CURRENT - { TARGET };

     repeat while(forall seq in CURRENT, seq[1] = TARGET[1]);
          PREFIX := PREFIX + <TARGET[1]>;
          TARGET := TARGET[2:];

          forall s in CURRENT;
               s := s[2:];
          end forall s;

     end repeat;

end FACTORS;


PSTARS(PUNIT): procedure;

PUNIT: P-unit; /* newly formed. */

/* An ordering is imposed on all atoms. This ordering
 is used for normalizing P-units. The lexicographic
 sorting routine has an obvious algorithm and
 is not shown. */

     PUNIT.SUBSEQUENCES := lex-sort(PUNIT.SUBSEQUENCES)

/* FINDTRUES is a simple routine which finds the
   collection of productions which were true on enter-
   ing the production contained in the first unit of
   the permutation. It is also capable of finding the
   same for a sequence of simple units or for the
   argument sequence of an R-unit. It is used in several
   places throughout the code and is not shown explicitly.*/

     TRPS := FINDTRUES(PUNIT);

/* If PSTARNAMES is empty, then we cannot compare
  this P-unit with any other.  We call P*FIXUP which
```

```
   generates new A-units. */

      if PSTARNAMES = {} then return
                                P*FIXUP(PUNIT,TRPS);
                                end if;

/* Lexico-equivalent is a Boolean function which
   compares two sequences for lexicographic equiva-
   lency.    Using lexico-equivalency allows for
   R-units. The code for lexico-equivalent is not
   shown.
    In what follows, we search PSTARNAMES for another
   P-unit which is equivalent to the one being
   formed. The canonizing makes this checking easier
   since we do not have to compare every pair of
   sequences. If one is found, both are replaced
   by a new A-unit which represents both. Notice,
   EQUIVSEQ is used to construct a 'merged'  seq-
   uence of each of the permutable subsequences.
   See the repetition routines for details of what
   EQUIVSEQ does. */

      if exists name in PSTARNAMES such that
         1 <= forall i <= #PUNIT.SUBSEQUENCES,
            lexico-equivalent(PUNIT.SUBSEQUENCES[i],
                              NAME.SUBSEQNC.SUBSEQUENCES[i])
       then
            1 <= forall j <= #PUNIT.SUBSEQUENCES;
                 PUNIT.SUBSEQUENCES[j] :=
                    EQUIVSEQ(PUNIT.SUBSEQUENCES[j],
                             1,
                             NAME.SUBSEQUENCES[j],
                             1,
                             CURRENTLN(NAME.
                                       SUBSEQUENCES[j]),
                             DUMMY);
            end forall j;

            NAME.SUBSEQNC := PUNIT;
            NAME.TRUE-PS := NAME.TRUE-PS + TRPS;
            NEWNM := P*FIXUP(PUNIT,NAME.TRUE-PS);
            NEWNM.EQUIVALENTTO := NAME;
            NEWNM.ORDERING := NAME.ORDERING;

            return NEWNM;

       end if;

/* If no equivalent A-unit was found, we construct a
   new one by calling P*FIXUP. */

      return P*FIXUP(PUNIT,TRPS);

 end PSTARS;
```

```
P*FIXUP(PUN,TRPS): procedure;

PUN: P-unit;
TRPS: set of atoms;

/* This routine generates new atom units. */

     NNAME := heap(A-unit);
     NNAME.SUBSEQNC := PUN;
     NNAME.TRUE-PS := TRPS;
     NNAME.ORDERING := PCOUNT;
     NNAME.EQUIVALENTTO := NNAME;
     PCOUNT := PCOUNT + 1;
     PSTARNAMES := PSTARNAMES + { NNAME };

     return NNAME;

end P*FIXUP;
```

2.3.3.4 Repetition Routines

The next process is the detection of repeating subsequences and replacement of such sequences by repetition units. Unlike the permutation detection problem, which is well-defined and solved by the algorithm described above, some sequences may have several equally acceptable descriptions which use the repetition operator. In fact, the corresponding minimization problem is NP-complete in the general case (see Section 1.4.2). The algorithm that we use gives one description. It employs a helpful heuristic with a 'divide and conquer' flavor.

The sequence is scanned to identify all single occurrences of A-units or rule names in S-units. All such occurrences cannot be part of a repetition and thus divide the sequence into shorter subsequences to which the same procedure can be applied recursively. Once a candidate subsequence is found (every atom in it occurs at least twice) a left to right scan is applied repeatedly to find equivalent adjacent subsequences (beginning with the shorter) and to replace them with R-units. The procedure stops when it considers sequences whose lengths are greater than one-half the current length of the sequence. The maximum length of subsequences we have to consider decreases when repetitions are constructed.

The length of a sequence differs from the length of the tuple encoding the sequence because of the appearance of

arbitrarily complex units. For example, the length of the following sequence as an n-tuple is 3 yet its length as a CRAPS sequence is 5: <<A (R.* 3 <<B C D>>(W.* ( ))(U.* ())) E>>.

The *while* and *until* components of R-units are computed from the TRUE-PS field of the S-units. A while condition is interpreted to be all of the active productions common to every entry of the repeating subsequence which are not active on exiting from the subsequence. An until condition is the opposite; everything that is active on exit but which was never active previously.

The critical aspect of this procedure is the equivalence of two sequences. Our interpretation is that the sequences must be lexicographically equal (the same names of A-units or production names of S-units appear in order) and aligned on a 'unit-boundary', that is, one sequence cannot start within an R-unit appearing at the end of the other. When two equivalent sequences are detected, a third sequence is constructed which is a merging of the R-units appearing in both.

For example,

<<A (R.* <<B>>(W.* ( )) (U.* (E))) C>>

and

<<A B (R.* <<C>> (W.* (G)) (U.* ( )))>>

are equivalent, and are merged to the sequence

<<A (R.* <<B>>(W.* ( ))(U.*(E))
(R.* <<C>> (W.* (G)) (U.* ( ))) >>.

Although merging sequences has the potential of generating many more control statements than we might want, we have found that the while and until conditions will keep this to a manageable number (however, it is possible that empty conditions are computed). In fact, whenever a merge of two sequences occurs, we compute new while and until conditions for any R-units that are merged, which reflects the conditions appearing in both units. This will mask out any erroneous iterations when the description is used in controlling a PS. See Figure 2.5 for the details of this entire process.

Figure 2.5. Repetition Detection.

```
REPEATS(SEQ,NXTUNIT): procedure;

SEQ: I-SEQUENCE;
NXTUNIT: I-unit; /* NXTUNIT is the unit which follows
            SEQ and which is used to calculate the
            WHILE and UNTIL conditions for any sub-
            sequences found repeating at the end of SEQ.*/

    if SEQ = omega then return omega; end if;

    NEWS := omega;
    TEST := SEQ[#SEQ];

/* Through a simple hashing scheme, the positions (ordered)
 of those units whose productions (or EQUIVALENTTO names
 in the case of A-units) appearing only once in SEQ,
 can be found easily.  We split SEQ into portions delimited
 by the unique units and recursively apply the same process
 to those portions. The FINDUNIQUES routine is not shown.*/
    UNIQUES := FINDUNIQUES(SEQ);
    if #UNIQUES = #SEQ then return SEQ; end if;
    if UNIQUES \= omega
     then
         repeat while(UNIQUES \= omega);
            FIRST := UNIQUES[1];
            UNIQUES := UNIQUES[2:];
            HOLD := SEQ[1:FIRST-1];
            FUNIT := SEQ[FIRST];
            SEQ := SEQ[FIRST+1:];
            NEWS := NEWS + REPEATS(HOLD,FUNIT)
                         + <FUNIT>;
         end repeat;

         if FUNIT = TEST then return NEWS;
         else return
                 NEWS + REPEATS(SEQ,NXTUNIT);
         end if;

    end if;

/* Everything in SEQ appears at least twice so we scan
  for repeating subsequences.  */

    for len := 1 by 1 repeat;
      if CURRENTLN(SEQ) / 2 < len then return SEQ; end if;

/* TRAIL, LEFT and RIGHT are scanning pointers. */

      TRAIL := 0;
      LEFT := 1;

/* ADVANCE positions a pointer some distance away from
  the first argument (another pointer). This distance
  is some number of units which encode a subsequence of
  length len (the second argument). The appearance of R-units
  makes this slightly difficult.  Consider the following:
```

< A, B, (R.* < C,A >),B,C >. When LEFT is 1 and len is 3 the pointer cannot be positioned because of the inner R-unit. Obviously, <A,B,C> is not a repeating subsequence because it is not aligned on a 'unit' boundary. The third argument to ADVANCE is set to true if the positioning problem occurs. ADVANCE returns omega as value if the positioning is possible but the remaining length of the sequence from that position is less than len.
The code for ADVANCE is simple. It uses a scan of the top level of the sequence keeping count of the cumulative length. When the length is equal to len it returns success of positioning but then goes on to test the remaining length. The code for ADVANCE is not shown. */

```
    repeat while( RIGHT \= omega ) doing (
                        RIGHT := ADVANCE(LEFT,len,FLAG,SEQ));

       if FLAG then /* the positioning cannot be done*/
          TRAIL := LEFT;
          LEFT := LEFT + 1;
          continue outerloop for len = 1 by 1;
       end if;

/* EQUIVSEQ returns a 'merge' sequence of two
  sequences if they are equivalent. It is explained
  in the following sections of code. */

       MERGE := EQUIVSEQ(SEQ,LEFT,SEQ,RIGHT,len,LASTPOS);

/* If we found a repeating subsequence we look for
  more by calling the next routine. All variables in
  this routine are global to the following call. */

       if MERGE \= omega
        then
         SUB-SCAN-LOOP();

         HOLD := SEQ[LASTPOS+1:];

/* COLLAPSE creates a new R-unit. INTSCT, UNYUN and
  LASTPOS get values from SUB-SCAN-LOOP.
   Notice in the call to COLLAPSE a conditional expression
  appears. HOLD equal to omega implies we found a
  repeating subsequence at the end of SEQ. Therefore
  we send NXTUNIT to COLLAPSE to compute the conditions
  for the new R-unit. Otherwise we just send the first
  unit of HOLD. */

         NEWSEQ := COLLAPSE(MERGE,INTSCT,UNYUN,len,
                         (if HOLD=omega then NXTUNIT
                          else HOLD[1] end if) );
         if TRAIL = 0 then
            SEQ := NEWSEQ + HOLD;
            TRAIL := 1;
            LEFT := 2;
         else
            SEQ := SEQ[1:TRAIL] + NEWSEQ + HOLD;
            LEFT := TRAIL + 2;
            TRAIL :=  TRAIL + 1;
         end if TRAIL;

    else /* MERGE is equal to omega */
        TRAIL := LEFT;
        LEFT := LEFT + 1;
```

```
        end if MERGE;

        end repeat while (RIGHT;

    end for len;

end REPEATS;


SUB-SCAN-LOOP(): procedure;

/* All variables within REPEATS are known here. */

     RYTE := LASTPOS + 1;

/* INTSCT and UNYUN collect the information needed to compute
the WHILE and UNTIL conditions. They simply accumulate the
intersection and union, respectively, of the set of FINDTRUES
for each instance of the repeating subsequence.
 LASTUNIT is a simple routine which returns the last unit
appearing in a subsequence of a certain length. It is a simple
scanning and counting algorithm which is not shown. */

     INTSCT := FINDTRUES(SEQ[LEFT]) * FINDTRUES(SEQ[RIGHT]);
     UNYUN := FINDTRUES(LASTUNIT(SEQ,LEFT,len)) +
                    FINDTRUES(LASTUNIT(SEQ,RIGHT,len));
     repeat while( MERGER \= omega )
       doing (
            MERGER := EQUIVSEQ(MERGE,1,SEQ,RYTE,len,ATLAST));
         MERGE := MERGER;
         LASTPOS := ATLAST;
         INTSCT := INTSCT *
                    FINDTRUES(SEQ[RYTE]);
         UNYUN := UNYUN +
                    FINDTRUES(LASTUNIT(SEQ,RYTE,len));
         RYTE := LASTPOS + 1;
     end repeat;

end SUB-SCAN-LOOP;


CURRENTLN(SEQ): procedure;

SEQ: I-SEQUENCE of S-, A- and R-units;

    CNTR := 0;

    1 <= forall i <= #SEQ;
        if SEQ[i] is an S-unit or A-unit then CNTR := CNTR + 1;
        else
           CNTR := CNTR + SEQ[i].LENTH;
        end if;
    end forall;

    return CNTR;
end CURRENTLN;
```

```
EQUIVSEQ(S1,P1,S2,P2,LEN,LAST): procedure;

S1,S2: I-SEQUENCES; /* Sequences being scanned for
                       equivalency. */
P1,P2: integer; /* pointers into S1 and S2 */
LEN: integer; /* length of subsequence being compared */
LAST: integer; /* pointer indicating last top level unit
                  in S2 scanned during the comparison. */

/* EQUIVSEQ is a recursive comparing routine. It compares
  two sequences, S1 and S2, at positions P1 and P2 of
  (CURRENTLN) length LEN and returns a merge sequence
  of the two if they were equivalent.  LAST is set to
  the last position scanned in S2 during the compare.
   The comparison procedes as follows:  Two S-units are
  equivalent if their PNAME's are equal. Two A-units
  are equivalent if they are EQUIVALENTTO the same A-
  unit. An R-unit is equivalent to some subsequence if
  its C-SEQ component is equivalent to that subsequence.
  Follow the code carefully for the R-unit comparison
  to see how the recursion works.
   The merged sequence returned is computed as follows:
  If two equivalent S- or A-units were detected, a copy
  of one of them is attached to the merged sequence. If
  an R-unit is detected to be equivalent to a subsequence,
  a copy of it is attached to the merged sequence after
  its C-SEQ component is replaced by the 'sub'-merged
  sequence produced by the recursive call to EQUIVSEQ.
  For example, given:
   <A,(R <B>), (R <C,D>)> and <A,B,(R <C>),D>, the routine
  would return the merged sequence
   <A,(R <B>),(R < (R <C>),D>) >.
   The REPLACE-W-U routine is used to merge the WHILE
  and UNTIL components of two equivalent R-units, merged
  together, by producing a DNF component computed by
  merging each of their corresponding DNF components.
  MERGEDNF does the same for simple units and is not shown.*/

     CNT := 0;
     MERGSEQ := omega;

     repeat while(CNT <= LEN);

        if P1 > #SEQ or P2 > #SEQ then return omega;
        end if;

        if SEQ[P1] is an A-unit and SEQ[P2] is an A-unit
         then
              if SEQ[P1].EQUIVALENTTO \= SEQ[P2].EQUIVALENTTO
               then return omega;
              else
               MERGSEQ := MERGSEQ + <SEQ[P1].EQUIVALENTTO>;
               CNT := CNT +1;
               LAST := P2;
               P1 := P1 + 1;
               P2 := P2 + 1;
              end if;

        else
         if S1[P1] is an S-unit and S2[P2] is an A-unit or
            S1[P1] is an A-unit and S2[P2] is an S-unit
          then return omega;

         else
```

```
if S1[P1] is an S-unit and S2[P2] is an S-unit
 then
  if S1[P1].PNAME \= S2[P2].PNAME
   then return omega;
  else
   MERGEDNF(S1[P1],S2[P2]);
   MERGSEQ := MERGSEQ + <S1[P1]>;
   CNT := CNT + 1;
   LAST := P2;
   P1 := P1 + 1;
   P2 := P2 + 1;
  end if;

else
 if S1[P1] is an R-unit
  then
      LC := S1[P1].LENTH;
      if S2[P2] is an R-unit
       then
           RC := S2[P2].LENTH;

           if RC = LC
            then
               HOLD := EQUIVSEQ(S1[P1].C-SEQ,
                                1,
                                S2[P2].C-SEQ,
                                1,
                                LC,
                                dummy);
               if HOLD = omega then return omega;
               end if;
               NEWUNIT := S1[P1];
               REPLACE-W-U(NEWUNIT,S2[P2],S2[P2]);
               NEWUNIT.C-SEQ := HOLD;
               MERGSEQ := MERGSEQ +  <NEWUNIT>;
               CNT := CNT + LC;
               LAST := P2;
               P1 := P1 + 1;
               P2 := P2 + 1;

           else
            if LC > RC
             then
                HOLD := EQUIVSEQ(S1[P1].C-SEQ,
                                 1,
                                 S2,
                                 P2,
                                 LC,
                                 LAST);
               if HOLD = omega then return omega;
               end if;
               NEWUNIT := S1[P1];
               NEWUNIT.C-SEQ := HOLD;
               REPLACE-W-U(NEWUNIT,
                           S2[P2],S2[LAST]);
               MERGSEQ := MERGSEQ + <NEWUNIT>;
               CNT := CNT + LC;
               P1 := P1 + 1;
               P2 := LAST + 1;

            else /* LC < RC */
               HOLD := EQUIVSEQ(S2[P2].C-SEQ,
                                1,
                                S1,
```

```
                                                    P1,
                                                    RC,
                                                    TEMP);
                                    if HOLD = omega then return omega;
                                    end if;
                                    NEWUNIT := S2[P2];
                                    NEWUNIT.C-SEQ := HOLD;
                                    REPLACE-W-U(NEWUNIT,
                                                S1[P1],S1[TEMP]);
                                    MERGSEQ := MERGSEQ + <NEWUNIT>;
                                    CNT := CNT + RC;
                                    P1 := TEMP + 1;
                                    LAST := P2;
                                    P2 := P2 + 1;

                                end if LC > RC;

                            end if LC = RC;

                        else /* S2[P2] is not an R-unit */
                            HOLD := EQUIVSEQ(S1[P1].C-SEQ,
                                             1,
                                             S2,
                                             P2,
                                             LC,
                                             LAST);
                            if HOLD = omega then return omega;
                            end if;
                            NEWUNIT := S1[P1];
                            NEWUNIT.C-SEQ := HOLD;
                            MERGSEQ := MERGSEQ + <NEWUNIT>;
                            P1 := P1 + 1;
                            P2 := LAST + 1;
                            CNT := CNT + LC;

                        end if S2[P2] is R-unit;

                    else /* S1[P1] is not an R-unit
                            but S2[P2] must be an R-unit. */
                         RC := S2[P2].LENTH;
                         HOLD := EQUIVSEQ(S1,P1,
                                          S2[P2].C-SEQ, 1,
                                          RC,DUMMY);
                         if HOLD = omega then return omega;
                         end if;
                         NEWUNIT := S2[P2];
                         NEWUNIT.C-SEQ := HOLD;
                         MERGSEQ := MERGSEQ + <NEWUNIT>;
                         CNT := CNT + RC;
                         LAST := P2;
                         P2 := P2 + 1;
                         P1 := P1 + RC;

                    end if S1[P1] is an R-unit;

            end repeat;

            if CNT = LEN then return MERGSEQ;
            else return omega;

        end EQUIVSEQ;
```

```
REPLACE-W-U(UNIT,WUNIT,UUNIT): procedure;

UNIT,UUNIT,WUINT: I-unit;

/* UNIT is an R-unit whose WHILE and UNTIL conditions
  will be merged with that of WUNIT and UUNIT. */

     UNIT.WHILE.DNF :=

          /* UNION-OR is a simple routine which merges
             two DNF components into one. It is not shown.*/

             UNION-OR(UNIT.WHILE.DNF,WUNIT.WHILE.DNF);

     if UUNIT is an R-unit then
          UNIT.UNTIL.DNF :=
               UNION-OR(UNIT.UNTIL.DNF,UUNIT.UNTIL.DNF);
     end if;

end REPLACE-W-U;

COLLAPSE(MERG,ENTER,ENDING,LEN,NXT): procedure;

MERGE: I-SEQUENCE; /* The merged sequence. */
ENTER: set of atoms; /* Rule names true on entering
                        the repeating subsequence. */
ENDING: set of atoms; /* Rule names true on leaving.*/
LEN: integer; /* Length of subsequence. */
NXT: I-unit; /* The unit which follows the repeating
                subsequence. */

/* This routine creates new R-units. */

     NEWUNIT := heap(R-unit);
     TRUEP := FINDTRUES(NXT);
     ENTER := ENTER - TRUEP;
     ENDING := TRUEP - ENDING;

/* The set of rule names is viewed as a simple conjunct. */

     NEWUNIT.LENTH := LEN;
     NEWUNIT.C-SEQ := MERGE;
     NEWUNIT.WHILE.DNF := ENTER;
     NEWUNIT.UNTIL.DNF := ENDING;

     return NEWUNIT;

end COLLAPSE;
```

2.3.3.5 Alternative Subsequence Detection

The final process is the coalescing of several descriptions of a set of solution sequences into one general description. Our construction is similar to that of Winston [49] and Hayes-Roth [19,20]. From two or more descriptions of some object we identify the points of similarity and alternate the differences.

This problem can be viewed as being equivalent to the Longest Common Subsequence problem. Since it is believed to be computationally intractable because of its NP-completeness (in the case of three or more sequences- Maier [29]), we do not intend to find the maximum number of points of similarity but rather a good heuristic approximation.

The set of descriptions is first ordered lexicographically to aid in discovering any equivalent ones which subsequently are replaced by one description. Then a target description is computed which is composed of a series of subsequences each common to all of the descriptions. This target identifies the points within a description that delimit smaller subsequences to be alternated. If the target is empty, a new target is computed. It proceeds by finding the 'best' series of subsequences common to any two descriptions. 'Best' is defined heuristically; more weight is given to A-units and R-units than S-units. If the target in this case is still empty the descriptions are collected into an ALT-unit with conditions computed as

described below.

Using the final computed target, all descriptions are scanned for the first occurrence of the common target subsequence. The initial portions of the descriptions scanned are then collected in a set to which the entire procedure is recursively applied. The procedure continues in this way until the target is exhausted. A special tag unit is appended to the end of each description to force the algorithm to completion.

A difficulty arises when a target subsequence does not appear in a particular description. In this case, a prefix subsequence must be selected so that it will be alternated with those prefix subsequences found in the descriptions which matched the target subsequence. This selection process is based on a simple heuristic: look for the first occurrence of a single unit that is either common to the target subsequence and nonmatching description, or common to one of the subsequences extracted from a matching description and the nonmatching description.

We demonstrate this process with an example. Given <<B D E>>, <<B D F G>> and <<D F G >>, the following events will occur: The target is computed to be < <<D>> <<T>> > since D is common to all and T is the special tag unit. Then we extract the prefixes <<B>>,<<B>> and << >>. The entire routine is recursively applied to these. The ordering produces <<>> and <<B>> as the new set of descriptions.

Since there is nothing common to both, the routine alternates them and returns <<(A.* (<< >>( )) (<<B>> (B)))>> to the top level. The sequence returned is concatenated with <<D>> to produce <<(A.* (<< >> ( )) (<<B>>(B))) D>>. Now <<T>> is the target. The prefixes collected this time are <<E>> <<F G>> and <<F G>>. Again the recursion orders them getting <<E>> and <<F G>>. No commonalities are detected so we return with <<(A.* (<<E>>(E)) (<<F G>>(F G)))>> The tag is removed from the sequence at the top level and the final description produced is <<(A.* (<< >>( ))(<<B>> (B))) D (A.* (<<E>>(E)) (<<F G>>(F G)))>>.

The computation for the conditions of the alternative subsequences is very simple. For each alternate, we find the set of rule names true on entry to each. From each of these sets we remove any production name appearing in any other. Intuitively, the conditions specify the productions which are true when a particular alternative is entered but which are not true on entering any other alternative. See Figure 2.6 for complete details of this entire process.

After computing alternation units, all A-units are replaced by the (recursively) cleansed versions of their corresponding P-units. All numeric components are removed (numbered PNAMEs and the lengths within the R-units). The final output is the CRAPS description.

Figure 2.6. Alternation Construction.

```
ALTERNATIVES(DESC): procedure;

DESC: set of I-SEQUENCE; /* All compiled descriptions. */

/* ORDER lexicographically orders the descriptions and merges
  any which are equivalent. It returns a sequence of descript-
  ions. It is a simple routine using lex-sort and EQUIVSEQ
  and is not shown. */

     DE := ORDER(DESC);
     if DE = omega then return DE; end if;
     if #DE = 1 then return DE[1]; end if;
     if omega is in DE then
        DE := DE - omega;
     end if;

/* COMMON computes a subsequence common to all descriptions.*/

     TARGET := COMMON(DE);
     if TARGET = omega then
        MAX := 0;

/* PAIR-LOOP is a routine which calls the function passed
  to it as second argument for every disjoint pair of objects
  contained in its first argument. The routine passed to it
  BEST-COM-SUBSEQ, will compute the best common subsequence
  found between any two descriptions. Notice TARGET and MAX
  are global to BEST-COM-SUBSEQ.
   The code for PAIR-LOOP is not shown. */

        PAIR-LOOP(DE,BEST-COM-SUBSEQ);

        if TARGET = omega /* no commonalities at all */ then
           return COALESCE(DE); /*which forms a new ALT-unit*/
        end if;

     end if; /* A series of common subsequences was found. */

/* To force the algorithm to completion, we tag all sequences
   with a dummy A-unit. */

     TARGET := TARGET + < <dummy-A-unit> >;

     1 <= forall i <= #DE;
        DE[i] := DE[i] + <dummy-A-unit>;
     end forall i;

     FINAL := omega;

     1 <= forall i <= #TARGET;
        MATCH,NONMATCH := {};
        NULLFLAG := false;
        CLCT := omega;

/* These two variables are used globally in FIX-UPS and are
  passed to other routines.  NXTONE is used for calculating
  the DNF expressions for alternatives. */
```

```
            NXTONE := TARGET[i+1][1];
            SEQ := TARGET[i];

            1 <= forall j <= #DE;

        /* FIND-FIRST-SEQ finds the first occurrence of the second
           argument sequence in the first argument sequence. It returns
           a vector of information. The first element is the beginning
           location of the sequence that was found. The second element
           is the position in the first sequence where it ended and the
           third element is the merged sequence combining the second
           argument and the matching portion of the first argument.
            It uses a scan with repeated calls to EQUIVSEQ. The code is
           not shown. */

                LOC := FIND-FIRST-SEQ(DE[j],SEQ);
                if LOC[1] = 0 then
                   NULLFLAG := true;
                   NONMATCH := NONMATCH + {j};
                else
                   MATCH := MATCH + {j};
                   HOLD := DE[j][1:LOC[2]];
                   DE[j] := DE[j][LOC[2]+1:];
                   CLCT := CLCT + <HOLD>;
                end if;
            end forall j;

        /* The following routine will handle the recursive calls to
           this routine after computing the appropriate prefixes from
           the non-matching descriptions. FINAL will be assigned values
           by FIX-UPS. */

               FIX-UPS();
           end forall i;

        /* We have to remove  the tag from FINAL. */

            FINAL := FINAL[1:#FINAL-1];
            return FINAL;

        end ALTERNATIVES;


        COMMON(DES): procedure;

        DES: sequence of descriptions.

            MAX := maxinteger;
            DES := sort-by-length-increasing(DES);
            HLD, TARGET := DES[1];

        /* The following loop will find a target description by
           continually removing subsequences from the target that
           are not common to all descriptions. */

            2 <= forall i <= #DES;

        /* TARGET will be adjusted to hold a sequence of
           subsequences found to be common to HLD and DES[i]
           by COM-SUBSEQ. Notice that < is sent as a function
           for the comparison in the routine. The subsequence
           with the least WEIGHT is necessarily the subsequence
           common to all descriptions. See COM-SUBSEQ. */
```

```
            COM-SUBSEQ(HLD,DES[i],<);
            if TARGET := omega then return omega; end if;
            HLD := omega;

            1 <= forall j <= #TARGET;

                /* The following just concatenates all
                  of the subsequences creating one sequence.*/

               HLD := HLD + TARGET[j];

            end forall j;

        end forall i;

        return TARGET;

end COMMON;


BEST-COM-SUBSEQ(X,Y): procedure;

X,Y: I-SEQUENCE;

/* We return the common subsequence with greatest WEIGHT. */

        return COM-SUBSEQ(X,Y,>);

end BEST-COM-SUBSEQ;

COM-SUBSEQ(X,Y,CFUNC): procedure, value(X,Y);

X,Y: I-SEQUENCE;
CFUNC: Boolean function;

        INIT := 0;
        LOCALWT := 0;
        LOCALTARGET := omega;

        repeat while(X \= omega and Y \= omega);

            1 <= forall i <= #X;
               LOC := FIND-FIRST-SEQ(Y,<X[i]>);
               if LOC[1] \= 0 then
                 INIT := i;
                 stop forall i;
               end if;
            end forall i;

            if INIT = 0 then return LOCALTARGET; end if;

    /* FIND-MAX-LENTH uses EQUIVSEQ to find the maximum length
       subsequence common to its two sequence arguments(first and
       third) rooted at the positions passed as second and fourth
       argument. It  is a simple scanning and counting routine
       and is not shown. */

             LEN := FIND-MAX-LENTH(X,INIT,Y,LOC[1]);
            MERGE := EQUIVSEQ(X,INIT,Y,LOC[1],LEN,LASTY);
             LOCALTARGET := LOCALTARGET + <MERGE>;

    /* WEIGHT is a heuristic weighting function which calculates
       the relative size of a sequence as follows:
```

```
     Each S-unit contributes 1, each R-unit contributes
    2 + the WEIGHT of its C-SEQ argument and A-units
    contribute 5 each.
     The routine is a simple scanning function and is not
    shown. */

          LOCALWT := LOCALWT + WEIGHT(MERGE);
          Y = Y[LASTY+1:];

     /* ADVANCE is discussed in REPEATS above.*/

          X := X[ADVANCE(X,LOC[1]+LEN):];

      end repeat;

/* Now we use the compare function: < is used for COMMON
   and > for BEST-COM-SUBSEQ.  */

      if CFUNC(LOCALWT,MAX) then
          MAX := LOCALWT;
          TARGET := LOCALTARGET;
      end if;

end COM-SUBSEQ;


FIX-UPS(): procedure;

      HLD := omega;

/* This routine will compute the appropriate prefixes for the
   recursive call to ALTERNATIVES by calling SUB-SELECTIONS and
   then it will assign the appropriate values to FINAL.
    All variables known in ALTERNATIVES are known here.
   NULLFLAG is true if there were descriptions not matching
   with the current TARGET subsequence. */

      if NULLFLAG then
         HLD := SUB-SELECTIONS(DE,CLCT,NONMATCH,MATCH,NXTONE);
      end if;

/* If there were non empty prefixes of the descriptions
   matching the current TARGET subsequence, we alternate them
   recursively. */

      if CLCT \= omega then
         FINAL := FINAL + ALTERNATIVES({CLCT[i] !
                                        1 <= forall i <= #CLCT});
      end if;

/* If every description had a subsequence matching the current
   TARGET subsequence, we append the TARGET subsequence to FINAL.
   Otherwise, we append an ALT-unit to FINAL. This unit is an
   alternation between the leading prefixes of the non-matches
   with the current target subsequence, or, an alternation of
   the current target subsequence with the null sequence
   depending upon whether prefixes were computed or not. */

          FINAL := FINAL + (if \NULLFLAG then SEQ
                            else if HLD \= omega then
                                   COALESCE(HLD + <SEQ>)
                                 else COALESCE(<SEQ,omega>)
                                 end if
                            end if);
```

```
end FIX-UPS;


SUB-SELECTIONS(DE,CL,NON,MAT,NXT): procedure;

DE: set of I-SEQUENCES; /* Descriptions. */
CL: sequence of I-SEQUENCES; /* Leading prefixes of the
                                matching descriptions. */
NON: set of integers; /* Indices of the nonmatches. */
MAT: set of integers; /* Indices of the matches. */
NXT: I-unit; /* First unit of the next target subsequence. */

/* This routine computes the set of prefixes of the non-matching
  descriptions which are to be used for the recursive call to
  ALTERNATIVES. */

        HLD := omega;
        FLAG := false;
     if CL \= omega then

        forall i in NON;

           1 <= forall j <= #CL;
              UNIT := CL[j][#CL[j]];
              LOC := FIND-FIRST-SEQ(DE[i],<UNIT>);
              if LOC[1] \= 0 then
                 CL := CL + <DE[i][1:LOC[1]-1]>;
                 DE[i] := DE[i][LOC[1]:];
                 NON := NON - {i};
                 continue forall i;
              end if;
           end forall j;

        end forall i;

     end if;

     forall k in NON;

        forall j in MAT;
           UNIT := DE[j][1];
           LOC := FIND-FIRST-SEQ(DE[k],<UNIT>);
           if LOC[1] \= 0 then
              HLD := HLD + <DE[k][1:LOC[1]-2]>;
              DE[k] := DE[k][LOC[1]-1:];
              continue forall k;
           end if;
        end forall j;

        FLAG := true;

     end forall k;

/* If we couldn't compute a prefix for one of the non-matches
  we use the null prefix. */

     if FLAG then
        HLD := HLD + <omega>;
     end if;

      return HLD;

end SUB-SELECTIONS;
```

```
COALESCE(DESC): procedure;

DESC: sequence of I-SEQUENCE;

/* This routine creates new ALT-units. */

/* The conditionals for each alternative is computed first.*/

     TOTALS, SUBTOT := omega;
     CLCTRUS := omega;

     l <= forall i <= #DESC;
        if DESC[i] = omega then
           CLCTRUS[i] := {};
        else
           CLCTRUS[i] := FINDTRUES(DESC[i][l]);
        end if;
     end forall i;

     l <= forall i <= #CLCTRUS;

          l <= forall j <= #CLCTRUS, j \= i;
             TOTAL[i] := TOTAL[i] + CLCTRUS[j];
          end forall j;

    end forall i;

    l <= forall i <= #CLCTRUS;
        CLCTRUS[i] := CLCTRUS[i] - TOTAL[i];
    end forall i;

    NEWUNIT := heap(ALT-unit);

    l <= forall i <= #DESC;
        NEWUNIT.COND[i].C-SEQ := DESC[i];
        NEWUNIT.COND[i].DNF := CLCTRUS[i];
    end forall i;

    return <NEWUNIT>;

end COALESCE;
```

2.3.3.6 Meta-Rule Construction

Construction of the meta-rules is based on the transition of the PS environment when firing a rule. The first type of meta-rule is concerned with how rules are activated. From the trace we can determine which rules may activate other productions when fired by computing the set difference of the new environment with the old. Similarly, for the second type of meta-rule (how rules are inactivated), we compute the set difference of the old environment with the new. The Currently-active type meta-rule is constructed by copying the environment in which a rule was fired, and the Just-fired type is constructed from the sequence of rules fired.

All meta-rules so constructed are collected in sets MADE-ACTIVE, MADE-INACTIVE, ENVIRONS and JUST-FIREDS, respectively. They are represented in the set by a pair <L,R>, where L is a set of production names corresponding to the argument of the LHS function and R, the RHS. Details appear in Figure 2.7.

Figure 2.7: Meta-rule Construction.

```
CONSTRUCT-M-RULE(LUNIT,RUNIT): procedure;

difnewold, difoldnew: set of atoms;
pair: tuple of sets;

/* Construction of meta-rules according to the definitions.*/

     difnewold := (RUNIT.TRUE-PS + {RUNIT.PNAME}) -
                  (LUNIT.TRUE-PS + {LUNIT.PNAME});

     difoldnew := (LUNIT.TRUE-PS + {LUNIT.PNAME}) -
                  (RUNIT.TRUE-PS + {RUNIT.PNAME});

/* The first element in the pairs corresponds to the LHS of
    a meta-rule, and the second to the RHS. */

/* Type 1 meta-rule... */

     if exists pair in MADE-ACTIVE such that
        pair[1] = difnewold then
         pair[2] := pair[2] + {LUNIT.PNAME};
        else
         MADE-ACTIVE := MADE-ACTIVE +
                        {<difnewold,{LUNIT.PNAME}>};
      end if;

/* Type 2 meta-rule... */

     if exists pair in MADE-INACTIVE such that
        pair[1] = difoldnew then
         pair[2] := pair[2] + {LUNIT.PNAME};
        else
         MADE-INACTIVE := MADE-INACTIVE +
                          {<difoldnew,{LUNIT.PNAME}>};
      end if;

/* Type 3 meta-rule... */

     if exists pair in ENVIRONS such that
        pair[1] = LUNIT.TRUE-PS  + {LUNIT.PNAME} then
         pair[2] := pair[2] + {LUNIT.PNAME};
        else
         ENVIRONS := ENVIRONS +
              {<LUNIT.TRUE-PS+{LUNIT.PNAME},{LUNIT.PNAME}>};
      end if;

/* Type 4 meta-rule... */

     if exists pair in JUST-FIREDS such that
        pair[1] = {LUNIT.PNAME} then
         pair[2] := pair[2] + {RUNIT.PNAME};
        else
         JUST-FIREDS := JUST-FIREDS +
                        {<{LUNIT.PNAME},{RUNIT.PNAME}>};
      end if;

end CONSTRUCT-M-RULE;
```

CHAPTER 3. EXPERIMENTS

In this chapter we discuss the experiments that were performed using the system and describe some of the insights we have gained from its behavior.

3.1 Jigsaw Puzzles

The Jigsaw Puzzle Production System (Section 1.2) consists of 33 productions containing an average of 3 data elements in the LHS, and was constructed in a few man hours. The representation was simplified so that knowledge of the physical locations of the puzzle pieces was ignored. This allowed us to concentrate on the more interesting problem of acquiring heuristics for solving problems rather than dealing with the complexities of moving objects in space.

The experiments were performed on 5 puzzles, each containing 25-30 pieces. Only one puzzle was used in the training session. The experiments showed that, on the average, 13 rules were applicable on each cycle, and with the size of the puzzles used, approximately 400 cycles were needed to solve a puzzle. In the absence of any heuristic controls, a production system such as the one in Section 1.2 would require an inordinate amount of time to search for a solution. (The reader is invited to compute the size of the search graph.) This situation appears to be an ideal one for demonstrating our approach. It seems likely that with

the introduction of simple sequencing rules, a system could solve a problem of this type in a reasonable amount of time and exhibit intelligent behavior. However, it does not appear likely that brute force, heuristic search methods, or more powerful representations would admit easy and general solutions to the jigsaw puzzle problem.

It can be argued that the productions we have defined can be encoded by just a few rules (perhaps 2 or 3). In this situation, the heuristics for solving this problem are trivial. However, such a representation would be highly inflexible and admit only limited or narrow behavior. We have attempted to define a set of productions which would allow for many solutions and many kinds of behavior. Further, many of the productions (in particular, the pile productions) are general enough to be used for other substantially different problems. It would seem possible then, that any heuristics we generate for these productions would be applicable in many different situations, and therefore would be of a more fundamentally general nature.

3.2 Execution with Training

The entire training session took several hours of connect time (mostly for bookkeeping) and 9 minutes of CPU time running the system on a CDC 6600 under UT LISP. The session was organized into a series of training sessions each intended to solve some subproblem of the overall problem. This allowed the human expert to experiment and decide on the most reasonable approach for demonstrating the solution of the problem. Furthermore, the limited resources available for the pattern recognition algorithms prohibited the analysis of solution sequences whose lengths were greater than 100 cycles. Each of the sequences produced from the shorter sessions were serially analyzed and the resulting descriptions concatenated to the CRAPS description contained in Appendix 2. Section 3.3 contains an abbreviated copy of the description.

The statistics produced by the PS interpreter for the complete training session is contained in Figure 3.1. Some of the statistics are concerned with the matching network produced by the PS interpreter from a compilation of the LHSs of the production memory. The output generated for this session is too large to reproduce here. However, we have included an abbreviated session of the fourth experiment running under CRAPS control in Appendix 3.

Figure 3.1 Training session statistics

```
(END -- EXPLICIT HALT)

33 PRODUCTIONS IN SYSTEM
127/300 NODES
391 PRODUCTIONS FIRED
3490 PRODUCTIONS INSTANTIATED
1808 WM TRANSACTIONS (8795 NODE ACTIVATIONS)
65200 TESTS PERFORMED
129 MAXIMUM WM SIZE
99.25831 MEAN WM SIZE
33 MAXIMUM CS SIZE
12.12788 MEAN CS SIZE
173 MAXIMUM NUMBER TOKENS STORED
113.75703 MEAN NUMBER TOKENS STORED


9.0 MINUTES CPU
```

3.3 Generated CRAPS Description and Meta-Rules

The CRAPS description (see Appendix 2) generated from the trace in Appendix 1, consists of 75 concatenations and 15 repetitions; no permutations were discovered, and no alternations were possible since only one sequence was analyzed. The DAG which was constructed contained many forward arcs which were correctly identified and discarded. An abbreviated version of the description is shown in Figure 3.2.

The fact that no permutations were generated may indicate the rather limited usefulness of such a construct. However, it appears more accurate to say that the solution presented to the system was produced from an environment in which parallelism was unlikely to occur. That is, the limitations of the communication channel between man and machine was not conducive to demonstrate parallel sequences of actions. At any one time a limited amount of information was displayed on the CRT screen and so the problem solution demonstrated was forced to be more narrow or localized decreasing the chances of parallel flows of information (while increasing the likelihood of short repeating subsequences). The problem of jigsaw puzzles would also appear to make limited use of parallelism since most of the activity is limited to a few puzzle pieces at any one time. However, many problems seem to require such a construct. For example, it is clear that an industrial robot with 2 (or more) arms operating on an assembly line would be required to perform

certain functions in a strict sequential fashion. However, its productivity would increase significantly if its arms were controlled concurrently.

Figure 3.2 Abbreviated Puzzle Heuristics

The preconditions of the simple units have been removed.

```
<<LOOK-AT-PIECE-IN-HEAP
  PICK-OBJECT-IN-VIEW
  MAKE-A-PILE
  [REPEAT (WHILE (LOOK-AT-PIECE-IN-HEAP))
          (UNTIL (EMPTY-HEAP))
    <<LOOK-AT-PIECE-IN-HEAP
      PICK-OBJECT-IN-VIEW
      PUT-OBJECT-IN-PILE>>]
  EMPTY-HEAP
  REMEMBER-CURRENT-PILE
  LOOK-AT-FIRST-IN-PILE
  REMEMBER-CURRENT-OBJECT
  LOOK-AT-NEXT-IN-PILE
  PICK-OBJECT-FROM-PILE
  FORGET-CURRENT-PILE
  MAKE-A-PILE
  FORGET-CURRENT-PILE
  PICK-A-PILE
  LOOK-AT-FIRST-IN-PILE
  [REPEAT (WHILE NIL)
          (UNTIL (PIECE-HAS-STRAIGHT-EDGE
                   REMEMBERED-OBJECT-IN-VIEW))
    <<[REPEAT (WHILE NIL)
              (UNTIL (PIECE-HAS-STRAIGHT-EDGE))
      <<LOOK-AT-NEXT-IN-PILE>>]
     [REPEAT (WHILE (PIECE-HAS-STRAIGHT-EDGE))
             (UNTIL (REMEMBERED-OBJECT-IN-VIEW))
       <<PICK-OBJECT-FROM-PILE
         FORGET-CURRENT-PILE
         PICK-A-PILE
         PUT-OBJECT-IN-PILE
         FORGET-CURRENT-PILE
         PICK-A-PILE
         LOOK-AT-FIRST-IN-PILE>>]>>]
  PICK-OBJECT-FROM-PILE
  FORGET-CURRENT-PILE
  PICK-A-PILE
  PUT-OBJECT-IN-PILE
  FORGET-REMEMBERED-OBJECT
  LOOK-AT-FIRST-IN-PILE
  PICK-OBJECT-FROM-PILE
  START-PUZZLE
  LOOK-AT-FIRST-IN-PILE
  [REPEAT (WHILE (LOOK-AT-NEXT-IN-PILE))
          (UNTIL NIL)
    <<[REPEAT (WHILE NIL)
              (UNTIL (PIECE-FITS-IN-PUZZLE))
      <<LOOK-AT-NEXT-IN-PILE>>]
     [REPEAT (WHILE (LOOK-AT-NEXT-IN-PILE
                      PIECE-FITS-IN-PUZZLE))
             (UNTIL NIL)
       <<PICK-OBJECT-FROM-PILE
         FIT-PIECE-IN-PUZZLE
         PIECE-PUT-IN-PUZZLE
         LOOK-AT-FIRST-IN-PILE>>]>>]
```

```
PICK-OBJECT-FROM-PILE
FIT-PIECE-IN-PUZZLE
PIECE-PUT-IN-PUZZLE
[REPEAT (WHILE NIL)
        (UNTIL (PUZZLE-IS-FINISHED))
  <<FORGET-CURRENT-PILE
    PICK-A-PILE
    LOOK-AT-FIRST-IN-PILE
    REMEMBER-CURRENT-OBJECT
    FIND-COLOR-OF-PIECE
    LOOK-AT-NEXT-IN-PILE
    [REPEAT (WHILE NIL)
            (UNTIL (REMEMBERED-OBJECT-IN-VIEW))
      <<[REPEAT (WHILE NIL)
                (UNTIL (PIECE-HAS-CURRENT-COLOR))
        <<LOOK-AT-NEXT-IN-PILE>>]
       [REPEAT (WHILE (FORGET-COLOR-OF-PIECE
                       PIECE-HAS-CURRENT-COLOR))
                (UNTIL (REMEMBERED-OBJECT-IN-VIEW))
        <<PICK-OBJECT-FROM-PILE
          FORGET-CURRENT-PILE
          PICK-A-PILE
          PUT-OBJECT-IN-PILE
          FORGET-CURRENT-PILE
          PICK-A-PILE
          LOOK-AT-FIRST-IN-PILE>>]>>]
    FORGET-REMEMBERED-OBJECT
    PICK-OBJECT-FROM-PILE
    FORGET-CURRENT-PILE
    PICK-A-PILE
    PUT-OBJECT-IN-PILE
    [REPEAT (WHILE LOOK-AT-FIRST-IN-PILE)
            (UNTIL (PILE-IS-EMPTY))
      <<LOOK-AT-FIRST-IN-PILE
        PICK-OBJECT-FROM-PILE
        FIT-PIECE-IN-PUZZLE
        PIECE-PUT-IN-PUZZLE>>]>>]
FORGET-CURRENT-PILE
[REPEAT (WHILE (PICK-A-PILE)
               FORGET-REMEMBERED-PILE))
        (UNTIL (THERE-ARE-NO-PILES))
  <<PICK-A-PILE
    DESTROY-A-PILE>>]
PUZZLE-IS-FINISHED>>
```

It is interesting to see exactly what heuristics are encoded by this description. The first 5 units:

```
<LOOK-AT-PIECE-IN-HEAP
 PICK-OBJECT-IN-VIEW
 MAKE-A-PILE
 [REPEAT (WHILE (LOOK-AT-PIECE-IN-HEAP))
                (UNTIL (EMPTY-HEAP))
    <LOOK-AT-PIECE-IN-HEAP
     PICK-OBJECT-IN-VIEW
     PUT-OBJECT-IN-PILE>]
EMPTY-HEAP
```

have the effect of ordering the pieces in the heap. It does so by picking (arbitrarily) a piece from the heap, putting it in a pile (MAKE-A-PILE), and repeating the same for the remaining pieces. Since the piles are in fact queues, the first piece placed in the pile will be the first piece to be viewed from the pile. The initial arbitrary selection of a piece in the heap is a source of error as we shall see shortly.

The next part of the sequence,

```
REMEMBER-CURRENT-PILE
LOOK-AT-FIRST-IN-PILE
REMEMBER-CURRENT-OBJECT
LOOK-AT-NEXT-IN-PILE
```

has the effect of initializing the pile for an ordered search. The first piece in the pile is tagged by REMEMBER-CURRENT-OBJECT and then placed at the end of the pile by LOOK-AT-NEXT-IN-PILE so that when it comes into view later, we will know that we have scanned the entire pile.
This is the point at which an error is introduced by the heuristics specified in the initial segment. The actual

preconditions of REMEMBER-CURRENT-OBJECT (see Appendix 2) specifies that PIECE-HAS-STRAIGHT-EDGE, that is, the first object in the pile, which is the first piece picked from the heap, must have a straight edge. Therefore, the initial selection of a piece from the heap should not be arbitrary, but instead should be one with a straight edge. This problem could have been avoided with additional training sequences for the initial subsequence and using the alternation routine to specify alternative actions. These errors are handled by the meta-rule mechanism.

The next part of the description,

```
PICK-OBJECT-FROM-PILE
FORGET-CURRENT-PILE
MAKE-A-PILE
FORGET-CURRENT-PILE
PICK-A-PILE
LOOK-AT-FIRST-IN-PILE
[REPEAT (WHILE NIL)
        (UNTIL (PIECE-HAS-STRAIGHT-EDGE
                REMEMBERED-OBJECT-IN-VIEW))
   <[REPEAT (WHILE NIL)
            (UNTIL (PIECE-HAS-STRAIGHT-EDGE))
      <LOOK-AT-NEXT-IN-PILE>]
    [REPEAT (WHILE (PIECE-HAS-STRIGHT-EDGE))
                   (UNTIL (REMEMBERED-OBJECT-IN-VIEW))
      <PICK-OBJECT-FROM-PILE
       FORGET-CURRENT-PILE
       PICK-A-PILE
       PUT-OBJECT-IN-PILE
       FORGET-CURRENT-PILE
       PICK-A-PILE
       LOOK-AT-FIRST-IN-PILE>]>]
PICK-OBJECT-FROM-PILE
FORGET-CURRENT-PILE
PICK-A-PILE
PUT-OBJECT-IN-PILE
FORGET-REMEMBERED-OBJECT
```

has the effect of creating a second pile composed of all of the pieces with straight edges. The initial pile is

repeatedly scanned (until the tagged piece is seen) and for each piece in view which has a straight edge, the piece is removed from the first pile and placed in the second. The appropriate conditions are encoded by the while and until components of the repetition units.

We now have one pile of indistinguished pieces and a second composed of all the straight edge pieces. We can now begin the puzzle by building the outside edges first from the pieces in the second pile until it is exhausted. The following subsequence performs these actions.

```
LOOK-AT-FIRST-IN-PILE
PICK-OBJECT-FROM-PILE
START-PUZZLE
LOOK-AT-FIRST-IN-PILE
[REPEAT (WHILE (LOOK-AT-NEXT-IN-PILE))
        (UNTIL NIL)
   <[REPEAT (WHILE NIL)
            (UNTIL (PIECE-FITS-IN-PUZZLE)
       <LOOK-AT-NEXT-IN-PILE>]
    [REPEAT (WHILE (LOOK-AT-NEXT-IN-PILE
                    PIECE-FITS-IN-PUZZLE))
            (UNTIL NIL)
       <PICK OBJECT-FROM-PILE
        FIT-PIECE-IN-PUZZLE
        PIECE-PUT-IN-PUZZLE

        LOOK-AT-FIRST-IN-PILE>]>]
PICK-OBJECT-FROM-PILE
FIT-PIECE-IN-PUZZLE
PIECE-PUT-IN-PUZZLE
```

Attention is now turned to the first pile. The following rather lengthy procedure is repeated until the first pile is exhausted in which case all of the puzzle pieces will have been placed in the puzzle.

If you study a children's puzzle, you will observe that (usually) all of the pieces with the same average

color are connected, and further, each color group is connected to the edge of the puzzle. This suggests the following procedure which was used by the trainer.
We look at the first piece in the first pile, note its color, tag it and place it at the end of the pile. Then we scan the entire pile, removing those pieces with the same average color and place them in a separate pile, called the color pile. This procedure is repeated until we encounter the tagged piece, which is removed and placed in the color pile. We then exhaustively scan the color pile, selecting the pieces that fit in the puzzle and placing them in the puzzle. The connectivity property of children's puzzles insures that this pile will be exhausted and no infinite loop through the pile is possible. Once this is accomplished, attention is turned back to the original pile, another color is selected and the entire process is repeated until the puzzle is finished. The following subsequence encodes the above heuristics.

```
[REPEAT (WHILE NIL)
        (UNTIL (PUZZLE-IS-FINISHED))
   <FORGET-CURRENT-PILE
    PICK-A-PILE
    LOOK-AT-FIRST-IN-PILE
    REMEMBER-CURRENT-OBJECT
    FIND-COLOR-OF-PIECE
    LOOK-AT-NEXT-IN-PILE
    [REPEAT (WHILE NIL)
            (UNTIL (REMEMBERED-OBJECT-IN-VIEW))
       <[REPEAT (WHILE NIL)
                (UNTIL (PIECE-HAS-CURRENT-COLOR))
          <LOOK-AT-NEXT-IN-PILE>]
        [REPEAT (WHILE (FORGET-COLOR-OF-PIECE
                        PIECE-HAS-CURRENT-COLOR))
                (UNTIL (REMEMBERED-OBJECT-IN-VIEW))
           <PICK-OBJECT-FROM-PILE
            FORGET-CURRENT-PILE
            PICK-A-PILE
            PUT-OBJECT-IN-PILE
            FORGET-CURRENT-PILE
            PICK-A-PILE
            LOOK-AT-FIRST-IN-PILE>]>]
    FORGET-REMEMBERED-OBJECT
    PICK-OBJECT-FROM-PILE
    FORGET-CURRENT-PILE
    PICK-A-PILE
    PUT-OBJECT-IN-PILE
    [REPEAT (WHILE (LOOK-AT-FIRST-IN-PILE))
            (UNTIL (PILE-IS-EMPTY))
       <LOOK-AT-FIRST-IN-PILE
        PICK-OBJECT-FROM-PILE
        FIT-PIECE-IN-PUZZLE
        PIECE-PUT-IN-PUZZLE>]>]
```

Now that all of the piles are exhausted and all of the pieces are in the puzzle, all of the piles are scanned in turn, destroying each of them until none remain :

```
FORGET-CURRENT-PILE
[REPEAT (WHILE (PICK-A-PILE
                FORGET-REMEMBERED-PILE))
        (UNTIL (THERE-ARE-NO-PILES))
   <PICK-A-PILE
    DESTROY-A-PILE>]
PUZZLE-IS-FINISHED>
```

Now the puzzle is finished. Appendix 3 contains an experiment using the above heuristics.

The analysis of the trace produced from the training session also produced 192 meta-rules: 47, 35, 90 and 20 of types 1, 2, 3 and 4 respectively. There are too many meta-rules to be reproduced here, but an arbitrary selection of a few of each type are shown in Figure 3.3.

Figure 3.3 Samples of the generated Meta-rules

```
Type 1 Meta-rules:

 [(<WANT-ACTIVE>
              (THERE-ARE-NO-PILES)
                   ) -->
                        (<TRY-TO-FIRE> (DESTROY-A-PILE))]
 [(<WANT-ACTIVE>
              (FORGET-CURRENT-PILE
               PILE-IS-EMPTY
               DESTROY-A-PILE
               REMEMBER-CURRENT-PILE)
                   ) -->
                        (<TRY-TO-FIRE> (PICK-A-PILE))]

Type 2 Meta-rules:

 [(<WANT-INACTIVE>
              (FORGET-REMEMBERED-PILE)
                   ) -->
                        (<TRY-TO-FIRE> (FORGET-REMEMBERED-PILE))]
 [(<WANT-INACTIVE>
              (CLOSE-EYES
               OBJECT-IN-HAND-IN-VIEW
               FIND-COLOR-OF-PIECE
               FORGET-COLOR-OF-PIECE
               PIECE-HAS-CURRENT-COLOR
               PIECE-FITS-IN-PUZZLE
               FIT-PIECE-IN-PUZZLE
               PIECE-PUT-IN-PUZZLE
               PUT-OBJECT-IN-PILE
               REMEMBER-CURRENT-OBJECT
               REMEMBERED-OBJECT-IN-VIEW
               REMEMBERED-OBJECT-IN-HAND)
                   ) -->
                        (<TRY-TO-FIRE> (PIECE-PUT-IN-PUZZLE))]

Type 3 Meta-rules:

 [(<CURRENTLY-ACTIVE>
              (PUZZLE-IS-FINISHED
               THERE-ARE-NO-PILES)
                   ) -->
                        (<TRY-TO-FIRE> (PUZZLE-IS-FINISHED))]
 [(<CURRENTLY-ACTIVE>
              (PUZZLE-IS-FINISHED
               THERE-ARE-NO-PILES
               FORGET-REMEMBERED-PILE)
                   ) -->
                        (<TRY-TO-FIRE>
                                  (FORGET-REMEMBERED-PILE))]

Type 4 Meta-rules:

 [(<JUST-FIRED> (LOOK-AT-PIECE-IN-HEAP))
                     -->
                        (<TRY-TO-FIRE>
                                  (PICK-UP-OBJECT-IN-VIEW))]
 [(<JUST-FIRED> (PICK-UP-OBJECT-IN-VIEW))
                     -->
                        (<TRY-TO-FIRE>
                                  (PUT-OBJECT-IN-PILE
                                   MAKE-A-PILE))]
```

The heuristics specified in the preceding pages are not general enough to solve every jigsaw puzzle. The human expert made certain assumptions that were reflected in the CRAPS description. In particular, when presented with a puzzle which does not have the connectivity property, it is possible that the system would fail to find a solution. However, the description is biased towards the solution demonstrated by the expert and so would exhibit intelligent behavior when presented with the appropriate inputs. A more general solution would be possible by conducting more training sessions, perhaps with different trainers, and using the alternation operator or the meta-rule mechanism. The meta-rules encode reasonable heuristics and correct statements of control in most cases. They are very different from those described by Davis and Kibler whose formalisms are highly data-dependent. Davis' meta-rules are acquired explicitly from a human expert and are composed of predicates applied to the data in WM. Kibler's meta-rules specify in effect 'data-flow links' between rules, and are compiled statically from production memory by an analysis of the contents of the LHS and RHS of rules. The rules we have constructed are dependent only on the conflict set of rules and the dynamic behavior of the program. This has the effect of limiting the suggested rules to those that were used in a solution. A large number of such rules are produced because of the large number of possible conflict sets contained in the solution sequence.

The meta-rules that have been defined here are flawed by their limited scope or context. The LHS of the meta-rules interrogate only the current conflict set or the previously fired production. In several similar situations, the rules MAKE-A-PILE and PICK-A-PILE are suggested with equal weight because of this limited scope. In this case, an arbitrary selection is not adequate and a deeper analysis of the situation is required to make the correct choice. More context within the solution sequence should be used to disambiguate the situation. We view our meta-rule construct as a starting point from which to analyze and understand the use of this type of mechanism. It is our belief that such rules exhibit adequate behavior when used in conjunction with the CRAPS descriptions. The primitives defined were derived from insights gained from several preliminary experiments with CRAPS. In view of the performance of the jigsaw puzzle PS, more research into the automatic construction of meta-rules is highly desirable. These points are discussed further in the following sections.

3.4 Execution with Heuristics

The original puzzle was easily solved under CRAPS control in 9.5 minutes with no meta-rule calls. The additional time required was due to the CRAPS implementation within the interpreter.

A second quite different puzzle was solved in 10.5 minutes but required 5 meta-rule calls. The situation described in the previous section (the first piece picked from the heap is required to have a straight edge) occurred and was correctly handled by the meta-rules. The first piece in the pile, although it did not have a straight edge, was tagged and eventually placed in the pile with the straight edge pieces. As the outside edges of the puzzle were being built, when the tagged piece was encountered and found to fit into the puzzle, the meta-rules suggested that it be placed in the puzzle. Subsequently, the CRAPS description worked correctly.

The following statistics were produced for this experiment.

```
(END -- EXPLICIT HALT)
(5 -- META RULE CALLS)
33 PRODUCTIONS IN SYSTEM
127/300 NODES
416 PRODUCTIONS-FIRED
3776 PRODUCTIONS-INSTANTIATED
1880 WM-TRANSACTIONS (70605 NODE ACTIVATIONS)
(69801 TESTS PERFORMED)
(137 MAXIMUM WM SIZE)
(107.32692 MEAN WH SIZE)
(35 MAXIMUM CS SIZE)
(13.12740 MEAN CS SIZE)
(181 MAXIMUM NUMBER TOKENS STORED)
(117.66587 MEAN NUMBER TOKENS STORED)
10.5 MINUTES CPU
```

The third puzzle failed to be solved because of an incorrect choice specified by the meta-rules. After 193 cycles, it had succeeded in ordering the heap and building the pile of straight edge pieces. The same situation occurred as described in the second experiment except that this time the tagged element did not fit in the puzzle when the outside edges were being built. At that point, the system looped through the pile containing the single tagged piece (all of the straight edge pieces had been placed in the puzzle). The repetition unit, which repeated LOOK-AT-NEXT-IN-PILE with the condition until PIECE-FITS-IN-PUZZLE never terminated.

The fourth puzzle worked properly. As you will note in Appendix 3, a very similar situation occurred as that described above. This time, the second piece in the pile, which is the focus of attention of the PICK-OBJECT-FROM-PILE unit following the REMEMBER-CURRENT-OBJECT unit, is also required by the preconditions to have a straight edge. In this case it did not. Rather than placing this piece in the pile of straight edge pieces, the meta-rules suggested the LOOK-AT-NEXT-IN-PILE production. This led to the correct choice of a piece from the pile (which had a straight edge) and subsequently the CRAPS description worked correctly.

A fifth puzzle was run without the CRAPS description and with the meta-rules alone. After 73 cycles and approximately 2.5 minutes, it had successfully ordered the heap by

piling the pieces. Immediately afterwards, the rules attempted to build a pile of straight edge pieces. At this time, both MAKE-A-PILE and PICK-A-PILE became equally likely candidates. The wrong choice was made, and the program proceeded to arbitrarily make piles in an almost hysterical fashion.

CHAPTER 4. CONCLUSIONS

4.1 Major Results

It is believed by many researchers that an important quality of intelligent behavior is the ability to improve performance with experience, and that generalizing a concept is a critical aspect of learning. In CRAPS, a form of generalization occurs when a repeating subsequence is collapsed into a repetition unit. Although quite restricted in scope, CRAPS shows how a system might learn procedures, a form of learning which is believed to be very important. Although there are a number of very difficult technical problems, it seems to us that with more powerful pattern recognition techniques and more powerful generalizations of control statements, this approach could be very fruitful.

With less ambitious designs, CRAPS can be viewed as a programming aid for the designer and implementer of a large AI problem-solving knowledge base. One of the most error prone and difficult tasks in the development of an AI problem-solving system is the fine-tuning of the system with heuristic controls to minimize search times through a large data base of facts. The CRAPS approach might be useful in fine-tuning a declarative knowledge base as opposed to 'hand-compiling' control elements to effect competent performance in such a system.

The power of the descriptions produced is limited by both the expressive power of the CRAPS primitives, and the level of sophistication of the pattern recognition algorithms that have been developed. For example, during repetition detection no notion of similar subsequence is used; furthermore, it is not possible for alternation to appear within repetitions. Despite this, the algorithms are powerful enough to detect interesting patterns and subsequently interesting heuristics.

Our experiments suggest that this approach will have the best chance of success when the encoding of the knowledge of the problem domain is such that on any execution cycle a small number of productions and only one instantiation of each production is applicable. For example, the initial segment of the CRAPS description for the jigsaw puzzle problem (see Appendix 2) specifies repeatedly picking a piece from the heap and placing it in a pile. In actuality, the number of instantiations of the LOOK-AT-PIECE-IN-HEAP production during this cycle is equal to the number of pieces currently in the heap. The CRAPS description does not specify which instantiation to choose; but only the name of the production to choose. This affects the subsequent ordering of the pieces in the pile, and possibly the preconditions of the units which follow in the description. This is demonstrated by the REMEMBER-CURRENT-OBJECT simple unit following the EMPTY-HEAP simple unit.

The preconditions for the unit specifies that the piece to remember (the first in the pile) should have a straight edge. But this depends on the first piece which was picked up from the heap, that is, the instantiation of PICK-UP-OBJECT-IN-VIEW that was selected. This suggests one way in which CRAPS could evolve: the execution traces should include not only productions but also instantiations of productions. Furthermore, the initial segmentation of the trace sequence into subsequences (or subproblems) specified by the human expert is not explicit in the final CRAPS description, and therefore contextual or goal information should be included. However, recognizing patterns in such sequences is a much more difficult problem. The approach outlined in this thesis is viewed as an initial step in the understanding of this more general problem.

The meta-rule construct we have defined seems quite effective. Although limited in scope, the meta-rules contain quite accurate and useful control information. The suggestion of a single rule in the RHS of a meta-rule effects a limited or local modification. More substantial statements of control can be effected by allowing arbitrary CRAPS descriptions in the RHS. However, a correspondingly more sophisticated analysis of both the solution sequence and CRAPS description would be required.

4.2 Future Research

There are many directions in which the present work can proceed. Pattern analysis is a long standing problem which obviously should be a continued research endeavor. The techniques outlined in this thesis leave much room for improvement and extension. The control language should be redefined to allow more powerful control primitives containing contextual or goal/subgoal information. An interesting question is the effect parallel machine architectures could have on the kind of analysis performed in this thesis.

The meta-rule construct requires more research in allowing for more powerful statements, such as CRAPS descriptions in the RHS and perhaps data-dependent conditions in the LHS. The implementation of the meta-rules, and in particular the selection algorithm for 'suggested' rules can be improved significantly by allowing for backtracking and other ordered search regimes.

With more powerful meta-rules, the correctness of the complete CRAPS description would be less critical; we might then hope to get successful performance with meta-rules alone, as suggested by the partial success on the fifth puzzle. However, this would probably require more understanding of some difficult context and control problems.

The existing system could be extended along the lines alluded to earlier. The utility and importance of both

counter-examples and data sequences (providing more specific knowledge about the problem domain) has been noticed by many researchers. The CRAPS system could be extended to include both types of information. Further, the existing system does not include the human engineering aspects of the type illustrated by Davis' system, and should be extended in this direction.

In Angluin [2,4], it has been shown that the pattern recognition problem attempted in this thesis is NP-complete. We have attempted to provide a good heuristic approximation for these problems. A theoretical basis of the feasibility of a more general approach is obviously needed. For instance, what is the necessary power (i.e. context free, regular expressions) of both the control language and the underlying knowledge representation model to effect competent performance in ill-defined problem domains (including problems for which no algorithmic procedure is known, such a computer vision).

In conclusion, the scope of the problems outlined in this thesis are of paramount importance to the field of artificial intelligence. Any small step made in solving the more general problems of course is significant for the entire field. We do not claim to have provided a solution, but we have identified a starting point, and a direction in which to proceed towards the solution of this enormously complex problem.

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APPENDIX 1

ABBREVIATED TRACE FOR PUZZLE PROBLEM.

THE SEQUENCE HAS BEEN CONDENSED. THE HUMAN EXPERT DIVIDED THE TRAINING SESSION INTO SEVERAL SESSIONS AS INDICATED BELOW. THE SUBSEQUENCES WERE SERIALLY ANALYZED, AND THE RESULTING (SUB) DESCRIPTIONS WERE CONCATENATED. ONE FINAL PASS OF THE REPETITION DETECTION ALGORITHM WAS APPLIED TO THIS SEQUENCE PRODUCING THE FINAL DESCRIPTION IN APPENDIX 2.

PRODUCTION NAME ABBREVIATION TABLE

LOOK-AT-PIECE-IN-HEAP	L-A-P-I-H
LOOK-AT-OBJECT-IN-HAND	L-A-O-I-H
CLOSE-EYES	C-E
OBJECT-IN-HAND-IN-VIEW	O-I-H-I-V
PICK-UP-OBJECT-IN-VIEW	P-U-O-I-V
PUT-PIECE-DOWN-IN-HEAP	P-P-D-I-H
EMPTY-HEAP	E-H
FIND-COLOR-OF-PIECE	F-C-O-P
FORGET-COLOR-OF-PIECE	FT-C-O-P
PIECE-HAS-STRAIGHT-EDGE	P-H-S-E
PIECE-HAS-CURRENT-COLOR	P-H-C-C
START-PUZZLE	S-P
PIECE-FITS-IN-PUZZLE	P-F-I-P
FIT-PIECE-IN-PUZZLE	F-P-I-P
PIECE-PUT-IN-PUZZLE	P-P-I-P
PUZZLE-IS-FINISHED	P-I-F
MAKE-A-PILE	M-A-P
PICK-A-PILE	P-A-P
PICK-OBJECT-FROM-PILE	P-O-F-P
PUT-OBJECT-IN-PILE	P-O-I-P
FORGET-CURRENT-PILE	F-C-P
PILE-IS-EMPTY	P-I-E
DESTROY-A-PILE	D-A-P
THERE-ARE-NO-PILES	T-A-N-P
LOOK-AT-FIRST-IN-PILE	L-A-F-I-P
LOOK-AT-NEXT-IN-PILE	L-A-N-I-P
REMEMBER-CURRENT-PILE	R-C-P
REMEMBERED-PILE-IS-CURRENT ...	R-P-I-C
FORGET-REMEMBERED-PILE	F-R-P
REMEMBER-CURRENT-OBJECT	R-C-O
FORGET-REMEMBERED-OBJECT	F-R-O
REMEMBERED-OBJECT-IN-VIEW	R-O-I-V
REMEMBERED-OBJECT-IN-HAND	R-O-I-H

TRACE SEQUENCE FOR PUZZLE PROBLEM
(PRODUCTION NAMES ARE ABBREVIATED)

EMPTY THE HEAP AND PILE THE PIECES:

```
<<<<<
  (L-A-P-I-H
     [L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      T-A-N-P ]
     (8 133)  (138 139))
  (P-U-O-I-V
     [L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      R-C-O F-C-O-P C-E T-A-N-P ]
     (137 139)  (140 145))
  (M-A-P
     [P-P-D-I-H S-P O-I-H-I-V L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H R-C-O F-C-O-P C-E
      T-A-N-P ]
     (143 144 145)  (146 155))
  (L-A-P-I-H
     [P-O-F-P L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      R-C-O F-C-O-P C-E D-A-P F-C-P R-C-P ]
     (150 151 152 153 154)  (156 157))
  (P-U-O-I-V
     [L-A-F-I-P P-H-S-E L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H R-C-O F-C-O-P C-E D-A-P F-C-P
      R-C-P ]
     (151 152 157)  (158 163))
  (P-O-I-P
     [P-P-D-I-H S-P O-I-H-I-V L-A-F-I-P P-H-S-E L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H R-C-O F-C-O-P
      C-E D-A-P F-C-P R-C-P ]
     (161 162 163)  (164 168))
  (L-A-P-I-H
     [L-A-F-I-P P-H-S-E L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H L-A-P-I-H
      R-C-O F-C-O-P C-E D-A-P F-C-P R-C-P ]
     (152 166 168)  (169 170))
  (P-U-O-I-V
```

```
      [L-A-F-I-F L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H
       L-A-F-I-H L-A-F-I-H L-A-F-I-H F-H-S-E L-A-F-I-H L-A-F-I-H
       L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H
       L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H
       R-C-O F-C-O-F C-E D-A-F F-C-F R-C-F ]
      (83 163 167 170)  (171 176))
   (F-O-I-F
      [F-F-D-I-H S-F O-I-H-I-V L-A-F-I-F L-A-F-I-H L-A-F-I-H
       L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H
       F-H-S-E L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H
       L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H L-A-F-I-H
       L-A-F-I-H L-A-F-I-H L-A-F-I-H R-C-O F-C-O-F C-E
       D-A-F F-C-F R-C-F ]
      (174 175 176)  (177 181))
                     .
                     .
                     .
   (THIS SEQUENCE REPEATED 22 TIMES)
                     .
                     .
                     .
   (E-H
      [L-A-F-I-F R-C-O F-C-O-F C-E D-A-F F-C-F
       R-C-F ]
      (152 465 467)  (468 468))
>>>>>
```

```
BUILD A PILE OF PIECES WITH STRAIGHT EDGES AND START PUZZLE
SET UP THE PILE FIRST:

<<<<<
  (R-C-P
     [L-A-F-I-P R-C-O F-C-O-P C-E D-A-P F-C-P ]
     (152 465 467)  (469 470))
  (L-A-F-I-P
     [R-P-I-C R-C-P F-R-P R-C-O F-C-O-P C-E
      D-A-P F-C-P ]
     (152 470)  (471 472))
  (R-C-O
     [P-O-F-P L-A-N-I-P P-H-S-E P-H-S-E F-C-O-P C-E
      R-P-I-C R-C-P F-R-P D-A-P F-C-P ]
     (152 153 465 466 472)  (473 474))
  (L-A-N-I-P
     [R-O-I-V R-C-O F-R-O P-O-F-P P-H-S-E P-H-S-E
      F-C-O-P C-E R-P-I-C R-C-P F-R-P D-A-P
      F-C-P ]
     (472 474)  (475 478))
  (P-O-F-P
     [L-A-N-I-P R-C-O P-H-S-E F-C-O-P C-E D-A-P
      F-R-O R-P-I-C R-C-P F-R-P F-C-P ]
     (152 153 466 477 478)  (479 483))
  (F-C-P
     [P-O-I-P S-P O-I-H-I-V L-A-F-I-P R-C-O P-H-S-E
      F-C-O-P C-E D-A-P F-R-O R-P-I-C R-C-P
      F-R-P ]
     (152 153 481 482 483)  (484 487))
  (M-A-P
     [P-A-P S-P P-H-S-E O-I-H-I-V R-C-O F-C-O-P
      C-E F-R-O F-R-P ]
     (155 482 483 486 487)  (488 497))
  (F-C-P
     [P-O-F-P R-C-O P-H-S-E F-C-O-P C-E D-A-P
      R-C-P F-R-O F-R-P ]
     (492 493 494 495 496)  (498 501))
  (P-A-P
     [R-C-O P-H-S-E F-C-O-P C-E F-R-O F-R-P ]
     (500 501)  (502 504))
  (L-A-F-I-P
     [D-A-P F-C-P R-P-I-C R-C-P R-C-O P-H-S-E
      F-C-O-P C-E F-R-O F-R-P ]
     (481 503 504)  (505 506))
>>>>>
```

NOW BUILD THE FILE:

```
<
(L-A-N-I-P
   [D-A-P P-C-P R-P-I-C R-C-P R-C-O P-C-O-P
    C-E P-R-O P-R-P ]
   (503 504 506)  (507 508))
(P-O-P-P
   [L-A-N-I-P R-C-O P-H-S-E P-C-O-P C-E D-A-P
    P-C-P R-P-I-C R-C-P P-R-O P-R-P ]
   (481 492 503 504 506)  (509 511))
(P-C-P
   [P-O-I-P S-P O-I-H-I-V R-C-O P-H-S-E P-C-O-P
    C-E D-A-P R-P-I-C R-C-P P-R-O P-R-P ]
   (503 504 509 510 511)  (512 515))
(P-A-P
   [M-A-P S-P O-I-H-I-V R-C-O P-H-S-E P-C-O-P
    C-E P-R-O P-R-P ]
   (497 510 511 514 515)  (516 518))
(P-O-I-P
   [D-A-P P-C-P L-A-P-I-P R-C-P S-P O-I-H-I-V
    R-C-O P-H-S-E P-C-O-P C-E P-R-O P-R-P ]
   (493 510 511 517 518)  (519 523))
(P-C-P
   [L-A-P-I-P R-C-O P-H-S-E P-C-O-P C-E D-A-P
    R-C-P P-R-O P-R-P ]
   (517 521 523)  (524 527))
(P-A-P
   [R-C-O P-H-S-E P-C-O-P C-E P-R-O P-R-P ]
   (526 527)  (528 530))
(L-A-P-I-P
   [D-A-P P-C-P R-P-I-C R-C-P R-C-O P-H-S-E
    P-C-O-P C-E P-R-O P-R-P ]
   (509 529 530)  (531 532))
                  .
                  .
                  .
(THIS SEQUENCE REPEATED 13 TIMES)
                  .
                  .
                  .
(P-O-P-P
   [L-A-N-I-P R-O-I-V R-C-O P-H-S-E P-H-S-E P-C-O-P
    C-E D-A-P P-C-P R-P-I-C R-C-P P-R-O
    P-R-P ]
   (892 899 900 905 906)  (907 911))
(P-C-P
   [P-O-I-P S-P O-I-H-I-V L-A-P-I-P R-O-I-V R-C-O
    P-H-S-E P-H-S-E P-C-O-P C-E R-O-I-H D-A-P
    R-P-I-C R-C-P P-R-O P-R-P ]
   (899 900 909 910 911)  (912 915))
(P-A-P
   [M-A-P S-P O-I-H-I-V R-O-I-V R-C-O P-H-S-E
    P-H-S-E P-C-O-P C-E R-O-I-H P-R-O P-R-P ]
   (497 910 911 914 915)  (916 918))
(P-O-I-P
   [D-A-P P-C-P L-A-P-I-P R-C-P S-P O-I-H-I-V
    R-O-I-V R-C-O P-H-S-E P-H-S-E P-C-O-P C-E
    R-O-I-H P-R-O P-R-P ]
```

```
    (891 910 911 917 918)  (919 923))
  (F-R-O
    [L-A-F-I-F R-O-I-V R-C-O F-H-S-E F-H-S-E F-C-O-F
     C-E D-A-F F-C-F R-C-F F-R-F ]
    (917 921 923)  (924 925))
  (L-A-F-I-F
    [R-C-O F-H-S-E F-H-S-E F-C-O-F C-E D-A-F
     F-C-F R-C-F F-R-F ]
    (923 925)  (926 929))
  (F-O-F-F
    [R-O-I-V R-C-O L-A-N-I-F F-H-S-E F-C-O-F C-E
     D-A-F F-C-F R-C-F F-R-F ]
    (927 929)  (930 934))
  (S-F
    [F-O-I-F O-I-H-I-V L-A-F-I-F R-C-O F-H-S-E F-C-O-F
     C-E D-A-F F-C-F R-C-F F-R-F ]
    (917 918 932 933 934)  (935 941))
  (L-A-F-I-F
    [D-A-F F-H-S-E F-C-F R-C-O R-C-F F-R-F
     F-C-O-F C-E ]
    (8 11 936)  (942 943))
>>:>>
```

```
BUILD OUTSIDE EDGES OF PUZZLE FIRST:

(L-A-N-I-P
   [P-O-F-P P-H-S-E R-C-O F-C-O-P C-E D-A-P
    F-C-P R-C-P F-R-P ]
   (6 8 11 113 943)  (944 947))
(P-O-F-P
   [L-A-N-I-P P-F-I-P P-H-S-E R-C-O F-C-O-P C-E
    D-A-P F-C-P R-C-P F-R-P ]
   (6 8 11 946 947)  (948 952))
(F-P-I-P
   [P-O-I-P O-I-H-I-V L-A-F-I-P P-F-I-P P-H-S-E R-C-O
    F-C-O-P C-E D-A-P F-C-P R-C-P F-R-P ]
   (8 11 950 951 952)  (953 957))
(P-P-I-P
   [F-P-I-P P-O-I-P O-I-H-I-V L-A-F-I-P P-F-I-P P-H-S-E
    R-C-O F-C-O-P C-E D-A-P F-C-P R-C-P
    F-R-P ]
   (114 951 952 957)  (958 965))
(L-A-F-I-P
   [D-A-P F-C-P R-C-P F-R-P ]
   (8 950 964)  (966 967))
                  .
                  .
                  .
(THIS SEQUENCE REPEATED 14 TIMES)
                  .
                  .
                  .
(P-O-F-P
   [P-F-I-P P-H-S-E P-H-S-E R-C-O F-C-O-P C-E
    D-A-P F-C-P R-C-P F-R-P ]
   (8 11 1282 1297 1299)  (1300 1304))
(F-P-I-P
   [P-O-I-P P-F-I-P O-I-H-I-V P-H-S-E P-H-S-E R-C-O
    F-C-O-P C-E D-A-P P-I-E F-C-P R-C-P
    F-R-P ]
   (8 11 1302 1303 1304)  (1305 1311))
(P-P-I-P
   [F-P-I-P P-O-I-P P-F-I-P O-I-H-I-V P-H-S-E P-H-S-E
    R-C-O F-C-O-P C-E D-A-P P-I-E F-C-P
    R-C-P F-R-P ]
   (1294 1303 1304 1311)  (1312 1319))
>>>>
```

SET UP A FILE:

```
<<<<
  (F-C-P
     [D-A-P P-I-E R-C-P F-R-P ]
     (8 11 1302)  (1320 1323))
  (P-A-P
     [F-R-P ]
     (1322 1323)  (1324 1326))
  (L-A-F-I-P
     [D-A-P F-C-P R-P-I-C R-C-P F-R-P ]
     (112 1325 1326)  (1327 1328))
  (R-C-O
     [P-O-F-P L-A-N-I-P F-C-O-P C-E D-A-P F-C-P
      R-P-I-C R-C-P F-R-P ]
     (112 1319 1325 1326 1328)  (1329 1330))
  (F-C-O-P
     [R-O-I-V R-C-O F-R-O P-O-F-P L-A-N-I-P C-E
      D-A-P F-C-P R-P-I-C R-C-P F-R-P ]
     (1328 1330)  (1331 1332))
  (L-A-N-I-P
     [P-H-C-C FT-C-O-P F-C-O-P R-O-I-V R-C-O F-R-O
      P-O-F-P C-C D-A-P F-C-P R-P-I-C R-C-P
      F-R-P ]
     (1328 1332)  (1333 1335))
>>>>
```

BUILD A PILE OF PIECES WITH THE SAME COLOR:

```
< :><
 (L-A-N-I-P
    [P-H-C-C FT-C-O-P F-C-O-P R-C-O F-R-O P-O-F-P
     C-E D-A-P F-C-P R-P-I-C R-C-P F-R-P ]
    (1334 1335)  (1336 1337))
 (P-O-F-P
    [L-A-N-I-P P-F-I-P P-H-C-C FT-C-O-P F-C-O-P R-C-O
     C-E D-A-P F-R-O F-C-P R-P-I-C R-C-P
     F-R-P ]
    (1319 1325 1326 1336 1337)  (1338 1341))
 (F-C-P
    [P-O-I-P F-P-I-P O-I-H-I-V L-A-F-I-P P-F-I-P P-H-C-C
     FT-C-O-P F-C-O-P R-C-O C-E D-A-P F-R-O
     R-P-I-C R-C-P F-R-P ]
    (1325 1326 1339 1340 1341)  (1342 1345))
 (P-A-P
    [M-A-P F-P-I-P O-I-H-I-V P-F-I-P P-H-C-C FT-C-O-P
     F-C-O-P R-C-O C-E F-R-O F-R-P ]
    (3 1340 1341 1344 1345)  (1346 1348))
 (P-O-I-P
    [D-A-P P-I-E F-C-P R-C-P F-P-I-P O-I-H-I-V
     P-F-I-P P-H-C-C FT-C-O-P F-C-O-P R-C-O C-E
     F-R-O F-R-P ]
    (1302 1340 1341 1347 1348)  (1349 1353))
 (F-C-P
    [P-O-F-P P-F-I-P P-H-C-C FT-C-O-P F-C-O-P R-C-O
     C-E D-A-P R-C-P F-R-O F-R-P ]
    (1347 1348 1351 1352 1353)  (1354 1357))
 (P-A-P
    [P-F-I-P P-H-C-C FT-C-O-P F-C-O-P R-C-O C-E
     F-R-O F-R-P ]
    (1356 1357)  (1358 1360))
 (L-A-F-I-P
    [D-A-P F-C-P R-P-I-C R-C-P P-F-I-P P-H-C-C
     FT-C-O-P F-C-O-P R-C-O C-E F-R-O F-R-P ]
    (1339 1359 1360)  (1361 1362))
                    .
                    .
                    .
(THIS SEQUENCE REPEATED 5 TIMES)
                    .
                    .
                    .
(F-R-O
    [P-O-F-P L-A-N-I-P P-H-C-C FT-C-O-P F-C-O-P R-O-I-V
     R-C-O C-E D-A-P F-C-P R-P-I-C R-C-P
     F-R-P ]
    (1473 1486 1493 1494 1496)  (1497 1498))
 (P-O-F-P
    [R-C-O L-A-N-I-P P-H-C-C FT-C-O-P F-C-O-P C-E
     D-A-P F-C-P R-P-I-C R-C-P F-R-P ]
    (1496 1498)  (1499 1503))
 (F-C-P
    [P-O-I-P O-I-H-I-V L-A-F-I-P R-C-O P-H-C-C FT-C-O-P
     F-C-O-P C-E D-A-P R-P-I-C R-C-P F-R-P ]
    (1493 1494 1501 1502 1503)  (1504 1507))
 (P-A-P
```

```
      [M-A-P O-I-H-I-V R-C-O P-H-C-C FT-C-O-P F-C-O-P
       C-E F-R-P ]
      (3 1502 1503 1506 1507)  (1508 1510))
   (P-O-I-P
      [D-A-P F-C-P L-A-F-I-P R-C-P O-I-H-I-V R-C-O
       P-H-C-C FT-C-O-P F-C-O-P C-E F-R-P ]
      (1485 1502 1503 1509 1510)  (1511 1515))
>>>>>
```

```
BUILD PUZZLE FROM PIECES WITH COMMON COLOR:

<<<
 (L-A-F-I-F
    [D-A-F F-C-F R-C-F F-R-F ]
    (1509 1513 1515)  (1516 1517))
 (F-O-F-F
    [L-A-N-I-F F-F-I-F R-C-O F-H-C-C FT-C-O-F F-C-O-F
     C-E D-A-F F-C-F R-C-F F-R-F ]
    (1509 1510 1513 1514 1517)  (1518 1522))
 (F-F-I-F
    [F-O-I-F O-I-H-I-V L-A-F-I-F F-F-I-F R-C-O F-H-C-C
     FT-C-O-F F-C-O-F C-E D-A-F F-C-F R-C-F
     F-R-F ]
    (1509 1510 1520 1521 1522)  (1523 1531))
 (F-F-I-F
    [F-F-I-F F-O-I-F O-I-H-I-V L-A-F-I-F F-F-I-F R-C-O
     F-H-C-C FT-C-O-F F-C-O-F C-E D-A-F F-C-F
     R-C-F F-R-F ]
    (1509 1521 1522 1531)  (1532 1539))
                    .
                    .
                    .
 (THIS SEQUENCE REPEATED 5 TIMES)
                    .
                    .
                    .
 (L-A-F-I-F
    [D-A-F F-C-F R-C-F F-R-F ]
    (1509 1634 1654)  (1656 1657))
 (F-O-F-F
    [F-F-I-F R-C-O F-H-C-C FT-C-O-F F-C-O-F C-E
     D-A-F F-C-F R-C-F F-R-F ]
    (1509 1510 1634 1655 1657)  (1658 1662))
 (F-F-I-F
    [F-O-I-F F-F-I-F O-I-H-I-V R-C-O F-H-C-C FT-C-O-F
     F-C-O-F C-E D-A-F F-I-E F-C-F R-C-F
     F-R-F ]
    (1509 1510 1660 1661 1662)  (1663 1671))
 (F-F-I-F
    [F-F-I-F F-O-I-F F-F-I-F O-I-H-I-V R-C-O F-H-C-C
     FT-C-O-F F-C-O-F C-E D-A-F F-I-E F-C-F
     R-C-F F-R-F ]
    (1652 1661 1662 1671)  (1672 1679))
>>>
```

```
                    .
                    .
                    .
WHAT FOLLOWED WAS A REPETITION OF THE PRECEEDING SEQUENCES:
       SET UP A FILE
       BUILD A FILE OF PIECES WITH SAME COLOR
       BUILD THE PUZZLE FROM THIS FILE
                    .
                    .
                    .


 FINALLY, CLEAN UP AND STOP!

<<<<<
  (F-C-P
     [P-I-F D-A-P P-I-E R-C-P F-R-P ]
     (8 11 1732)  (1786 1789))
  (P-A-P
     [P-I-F F-R-P ]
     (1732 1787)  (1790 1792))
  (D-A-P
     [P-I-F F-C-P P-I-E R-C-P F-R-P ]
     (1787 1791 1792)  (1793 1797))
  (P-A-P
     [P-I-F F-R-P ]
     (3 1794 1796)  (1798 1800))
  (D-A-P
     [P-I-F P-I-E F-C-P R-C-P F-R-P ]
     (2 1799 1800)  (1801 1805))
  (F-R-P
     [T-A-N-P P-I-F ]
     (470 1804 1805)  (1806 1807))
  (P-I-F
     [T-A-N-P ]
     (1804 1805)  (1808 1808))
>>>>>


       *** END OF TRACE ***
```

APPENDIX 2

PRETTY PRINTED CRAPS OUTPUT FOR PUZZLE PS.

```
<<
 (LOOK-AT-PIECE-IN-HEAP
      (THERE-ARE-NO-PILES))
 (PICK-UP-OBJECT-IN-VIEW
      (LOOK-AT-PIECE-IN-HEAP
      CLOSE-EYES
      FIND-COLOR-OF-PIECE
      THERE-ARE-NO-PILES
      REMEMBER-CURRENT-OBJECT))
 (MAKE-A-PILE
      (LOOK-AT-PIECE-IN-HEAP
      CLOSE-EYES
      OBJECT-IN-HAND-IN-VIEW
      PUT-PIECE-DOWN-IN-HEAP
      FIND-COLOR-OF-PIECE
      START-PUZZLE
      THERE-ARE-NO-PILES
      REMEMBER-CURRENT-OBJECT))
 [REPEAT     (WHILE  (LOOK-AT-PIECE-IN-HEAP))
             (UNTIL  (EMPTY-HEAP))
    <<
     (LOOK-AT-PIECE-IN-HEAP
          (OR.*
          (CLOSE-EYES
           FIND-COLOR-OF-PIECE
           PIECE-HAS-STRAIGHT-EDGE
           FORGET-CURRENT-PILE
           DESTROY-A-PILE
           REMEMBER-CURRENT-PILE
           REMEMBER-CURRENT-OBJECT)
          (CLOSE-EYES
           FIND-COLOR-OF-PIECE
           FORGET-CURRENT-PILE
           DESTROY-A-PILE
           REMEMBER-CURRENT-PILE
           REMEMBER-CURRENT-OBJECT)
          (CLOSE-EYES
           FIND-COLOR-OF-PIECE
           PIECE-HAS-STRAIGHT-EDGE
           PICK-OBJECT-FROM-PILE
           FORGET-CURRENT-PILE
           DESTROY-A-PILE
           REMEMBER-CURRENT-PILE
           REMEMBER-CURRENT-OBJECT)))
      (PICK-UP-OBJECT-IN-VIEW
          (OR.*
          (CLOSE-EYES
           FIND-COLOR-OF-PIECE
           FORGET-CURRENT-PILE
           DESTROY-A-PILE
           LOOK-AT-FIRST-IN-PILE
           REMEMBER-CURRENT-PILE
           REMEMBER-CURRENT-OBJECT)
```

```
          (LOOK-AT-PIECE-IN-HEAP
           CLOSE-EYES
           FIND-COLOR-OF-PIECE
           PIECE-HAS-STRAIGHT-EDGE
           FORGET-CURRENT-FILE
           DESTROY-A-FILE
           LOOK-AT-FIRST-IN-FILE
           REMEMBER-CURRENT-FILE
           REMEMBER-CURRENT-OBJECT)
          (LOOK-AT-PIECE-IN-HEAP
           CLOSE-EYES
           FIND-COLOR-OF-PIECE
           FORGET-CURRENT-FILE
           DESTROY-A-FILE
           LOOK-AT-FIRST-IN-FILE
           REMEMBER-CURRENT-FILE
           REMEMBER-CURRENT-OBJECT)))
    (PUT-OBJECT-IN-FILE
          (OR.*
          (CLOSE-EYES
           OBJECT-IN-HAND-IN-VIEW
           PUT-PIECE-DOWN-IN-HEAP
           EMPTY-HEAP
           FIND-COLOR-OF-PIECE
           START-PUZZLE
           FORGET-CURRENT-FILE
           DESTROY-A-FILE
           LOOK-AT-FIRST-IN-FILE
           REMEMBER-CURRENT-FILE
           REMEMBER-CURRENT-OBJECT)
          (LOOK-AT-PIECE-IN-HEAP
           CLOSE-EYES
           OBJECT-IN-HAND-IN-VIEW
           PUT-PIECE-DOWN-IN-HEAP
           FIND-COLOR-OF-PIECE
           PIECE-HAS-STRAIGHT-EDGE
           START-PUZZLE
           FORGET-CURRENT-FILE
           DESTROY-A-FILE
           LOOK-AT-FIRST-IN-FILE
           REMEMBER-CURRENT-FILE
           REMEMBER-CURRENT-OBJECT)
          (LOOK-AT-PIECE-IN-HEAP
           CLOSE-EYES
           OBJECT-IN-HAND-IN-VIEW
           PUT-PIECE-DOWN-IN-HEAP
           FIND-COLOR-OF-PIECE
           START-PUZZLE
           FORGET-CURRENT-FILE
           DESTROY-A-FILE
           LOOK-AT-FIRST-IN-FILE
           REMEMBER-CURRENT-FILE
           REMEMBER-CURRENT-OBJECT)))
   >>
]
(EMPTY-HEAP
     (CLOSE-EYES
     FIND-COLOR-OF-PIECE
     FORGET-CURRENT-FILE
     DESTROY-A-FILE
```

```
     LOOK-AT-FIRST-IN-FILE
     REMEMBER-CURRENT-FILE
     REMEMBER-CURRENT-OBJECT))
(REMEMBER-CURRENT-FILE
     (CLOSE-EYES
     FIND-COLOR-OF-PIECE
     FORGET-CURRENT-FILE
     DESTROY-A-FILE
     LOOK-AT-FIRST-IN-FILE
     REMEMBER-CURRENT-OBJECT))
(LOOK-AT-FIRST-IN-FILE
     (CLOSE-EYES
     FIND-COLOR-OF-PIECE
     FORGET-CURRENT-FILE
     DESTROY-A-FILE
     REMEMBER-CURRENT-FILE
     REMEMBERED-FILE-IS-CURRENT
     FORGET-REMEMBERED-FILE
     REMEMBER-CURRENT-OBJECT))
(REMEMBER-CURRENT-OBJECT
     (CLOSE-EYES
     FIND-COLOR-OF-PIECE
     PIECE-HAS-STRAIGHT-EDGE
     PICK-OBJECT-FROM-FILE
     FORGET-CURRENT-FILE
     DESTROY-A-FILE
     LOOK-AT-NEXT-IN-FILE
     REMEMBER-CURRENT-FILE
     REMEMBERED-FILE-IS-CURRENT
     FORGET-REMEMBERED-FILE))
(LOOK-AT-NEXT-IN-FILE
     (CLOSE-EYES
     FIND-COLOR-OF-PIECE
     PIECE-HAS-STRAIGHT-EDGE
     PICK-OBJECT-FROM-FILE
     FORGET-CURRENT-FILE
     DESTROY-A-FILE
     REMEMBER-CURRENT-FILE
     REMEMBERED-FILE-IS-CURRENT
     FORGET-REMEMBERED-FILE
     REMEMBER-CURRENT-OBJECT
     FORGET-REMEMBERED-OBJECT
     REMEMBERED-OBJECT-IN-VIEW))
(PICK-OBJECT-FROM-FILE
     (CLOSE-EYES
     FIND-COLOR-OF-PIECE
     PIECE-HAS-STRAIGHT-EDGE
     FORGET-CURRENT-FILE
     DESTROY-A-FILE
     LOOK-AT-NEXT-IN-FILE
     REMEMBER-CURRENT-FILE
     REMEMBERED-FILE-IS-CURRENT
     FORGET-REMEMBERED-FILE
     REMEMBER-CURRENT-OBJECT
     FORGET-REMEMBERED-OBJECT))
(FORGET-CURRENT-FILE
     (CLOSE-EYES
     OBJECT-IN-HAND-IN-VIEW
     FIND-COLOR-OF-PIECE
     PIECE-HAS-STRAIGHT-EDGE
```

```
      START-PUZZLE
      PUT-OBJECT-IN-PILE
      DESTROY-A-PILE
      LOOK-AT-FIRST-IN-PILE
      REMEMBER-CURRENT-PILE
      REMEMBERED-PILE-IS-CURRENT
      FORGET-REMEMBERED-PILE
      REMEMBER-CURRENT-OBJECT
      FORGET-REMEMBERED-OBJECT))
(MAKE-A-PILE
      (CLOSE-EYES
      OBJECT-IN-HAND-IN-VIEW
      FIND-COLOR-OF-PIECE
      START-PUZZLE
      PICK-A-PILE
      FORGET-REMEMBERED-PILE
      REMEMBER-CURRENT-OBJECT
      FORGET-REMEMBERED-OBJECT))
(FORGET-CURRENT-PILE
      (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      PIECE-HAS-STRAIGHT-EDGE
      PICK-OBJECT-FROM-PILE
      DESTROY-A-PILE
      REMEMBER-CURRENT-PILE
      FORGET-REMEMBERED-PILE
      REMEMBER-CURRENT-OBJECT
      FORGET-REMEMBERED-OBJECT))
(PICK-A-PILE
      (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      PIECE-HAS-STRAIGHT-EDGE
      FORGET-REMEMBERED-PILE
      REMEMBER-CURRENT-OBJECT
      FORGET-REMEMBERED-OBJECT))
(LOOK-AT-FIRST-IN-PILE
      (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      PIECE-HAS-STRAIGHT-EDGE
      FORGET-CURRENT-PILE
      DESTROY-A-PILE
      REMEMBER-CURRENT-PILE
      REMEMBERED-PILE-IS-CURRENT
      FORGET-REMEMBERED-PILE
      REMEMBER-CURRENT-OBJECT
      FORGET-REMEMBERED-OBJECT))
[REPEAT     (WHILE  NIL)
            (UNTIL  (PIECE-HAS-STRAIGHT-EDGE
                     REMEMBERED-OBJECT-IN-VIEW))
   <<
    [REPEAT     (WHILE  NIL)
                (UNTIL  (PIECE-HAS-STRAIGHT-EDGE))
       <<
        (LOOK-AT-NEXT-IN-PILE
             (CLOSE-EYES
             FIND-COLOR-OF-PIECE
             PICK-OBJECT-FROM-PILE
             FORGET-CURRENT-PILE
             DESTROY-A-PILE
             REMEMBER-CURRENT-PILE
```

```
          REMEMBERED-FILE-IS-CURRENT
          FORGET-REMEMBERED-FILE
          REMEMBER-CURRENT-OBJECT
          FORGET-REMEMBERED-OBJECT))
   >>
]
[REPEAT    (WHILE  (PIECE-HAS-STRAIGHT-EDGE))
           (UNTIL  (REMEMBERED-OBJECT-IN-VIEW))
   <-
    (PICK-OBJECT-FROM-FILE
          (CLOSE-EYES
          FIND-COLOR-OF-PIECE
          PIECE-HAS-STRAIGHT-EDGE
          FORGET-CURRENT-FILE
          DESTROY-A-FILE
          LOOK-AT-NEXT-IN-FILE
          REMEMBER-CURRENT-FILE
          REMEMBERED-FILE-IS-CURRENT
          FORGET-REMEMBERED-FILE
          REMEMBER-CURRENT-OBJECT
          FORGET-REMEMBERED-OBJECT))
    (FORGET-CURRENT-FILE
          (CLOSE-EYES
          OBJECT-IN-HAND-IN-VIEW
          FIND-COLOR-OF-PIECE
          PIECE-HAS-STRAIGHT-EDGE
          START-PUZZLE
          PUT-OBJECT-IN-FILE
          DESTROY-A-FILE
          LOOK-AT-FIRST-IN-FILE
          REMEMBER-CURRENT-FILE
          REMEMBERED-FILE-IS-CURRENT
          FORGET-REMEMBERED-FILE
          REMEMBER-CURRENT-OBJECT
          FORGET-REMEMBERED-OBJECT))
    (PICK-A-FILE
          (CLOSE-EYES
          OBJECT-IN-HAND-IN-VIEW
          FIND-COLOR-OF-PIECE
          PIECE-HAS-STRAIGHT-EDGE
          START-PUZZLE
          MAKE-A-FILE
          FORGET-REMEMBERED-FILE
          REMEMBER-CURRENT-OBJECT
          FORGET-REMEMBERED-OBJECT))
    (PUT-OBJECT-IN-FILE
          (CLOSE-EYES
          OBJECT-IN-HAND-IN-VIEW
          FIND-COLOR-OF-PIECE
          PIECE-HAS-STRAIGHT-EDGE
          START-PUZZLE
          FORGET-CURRENT-FILE
          DESTROY-A-FILE
          LOOK-AT-FIRST-IN-FILE
          REMEMBER-CURRENT-FILE
          FORGET-REMEMBERED-FILE
          REMEMBER-CURRENT-OBJECT
          FORGET-REMEMBERED-OBJECT))
    (FORGET-CURRENT-FILE
          (CLOSE-EYES
```

```
                FIND-COLOR-OF-PIECE
                PIECE-HAS-STRAIGHT-EDGE
                DESTROY-A-FILE
                LOOK-AT-FIRST-IN-FILE
                REMEMBER-CURRENT-FILE
                FORGET-REMEMBERED-FILE
                REMEMBER-CURRENT-OBJECT
                FORGET-REMEMBERED-OBJECT))
          (PICK-A-FILE
                (CLOSE-EYES
                FIND-COLOR-OF-PIECE
                PIECE-HAS-STRAIGHT-EDGE
                FORGET-REMEMBERED-FILE
                REMEMBER-CURRENT-OBJECT
                FORGET-REMEMBERED-OBJECT))
          (LOOK-AT-FIRST-IN-FILE
                (CLOSE-EYES
                FIND-COLOR-OF-PIECE
                PIECE-HAS-STRAIGHT-EDGE
                FORGET-CURRENT-FILE
                DESTROY-A-FILE
                REMEMBER-CURRENT-FILE
                REMEMBERED-FILE-IS-CURRENT
                FORGET-REMEMBERED-FILE
                REMEMBER-CURRENT-OBJECT
                FORGET-REMEMBERED-OBJECT))
          >>
     ]
    >>
]
(PICK-OBJECT-FROM-FILE
      (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      PIECE-HAS-STRAIGHT-EDGE
      FORGET-CURRENT-FILE
      DESTROY-A-FILE
      LOOK-AT-NEXT-IN-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
      FORGET-REMEMBERED-FILE
      REMEMBER-CURRENT-OBJECT
      FORGET-REMEMBERED-OBJECT
      REMEMBERED-OBJECT-IN-VIEW))
(FORGET-CURRENT-FILE
      (CLOSE-EYES
      OBJECT-IN-HAND-IN-VIEW
      FIND-COLOR-OF-PIECE
      PIECE-HAS-STRAIGHT-EDGE
      START-PUZZLE
      PUT-OBJECT-IN-FILE
      DESTROY-A-FILE
      LOOK-AT-FIRST-IN-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
      FORGET-REMEMBERED-FILE
      REMEMBER-CURRENT-OBJECT
      FORGET-REMEMBERED-OBJECT
      REMEMBERED-OBJECT-IN-VIEW
      REMEMBERED-OBJECT-IN-HAND))
(PICK-A-FILE
```

```
     (CLOSE-EYES
     OBJECT-IN-HAND-IN-VIEW
     FIND-COLOR-OF-PIECE
     PIECE-HAS-STRAIGHT-EDGE
     START-PUZZLE
     MAKE-A-FILE
     FORGET-REMEMBERED-FILE
     REMEMBER-CURRENT-OBJECT
     FORGET-REMEMBERED-OBJECT
     REMEMBERED-OBJECT-IN-VIEW
     REMEMBERED-OBJECT-IN-HAND))
(PUT-OBJECT-IN-FILE
     (CLOSE-EYES
     OBJECT-IN-HAND-IN-VIEW
     FIND-COLOR-OF-PIECE
     PIECE-HAS-STRAIGHT-EDGE
     START-PUZZLE
     FORGET-CURRENT-FILE
     DESTROY-A-FILE
     LOOK-AT-FIRST-IN-FILE
     REMEMBER-CURRENT-FILE
     FORGET-REMEMBERED-FILE
     REMEMBER-CURRENT-OBJECT
     FORGET-REMEMBERED-OBJECT
     REMEMBERED-OBJECT-IN-VIEW
     REMEMBERED-OBJECT-IN-HAND))
(FORGET-REMEMBERED-OBJECT
     (CLOSE-EYES
     FIND-COLOR-OF-PIECE
     PIECE-HAS-STRAIGHT-EDGE
     FORGET-CURRENT-FILE
     DESTROY-A-FILE
     LOOK-AT-FIRST-IN-FILE
     REMEMBER-CURRENT-FILE
     FORGET-REMEMBERED-FILE
     REMEMBER-CURRENT-OBJECT
     REMEMBERED-OBJECT-IN-VIEW))
(LOOK-AT-FIRST-IN-FILE
     (CLOSE-EYES
     FIND-COLOR-OF-PIECE
     PIECE-HAS-STRAIGHT-EDGE
     FORGET-CURRENT-FILE
     DESTROY-A-FILE
     REMEMBER-CURRENT-FILE
     FORGET-REMEMBERED-FILE
     REMEMBER-CURRENT-OBJECT))
(PICK-OBJECT-FROM-FILE
     (CLOSE-EYES
     FIND-COLOR-OF-PIECE
     PIECE-HAS-STRAIGHT-EDGE
     FORGET-CURRENT-FILE
     DESTROY-A-FILE
     LOOK-AT-NEXT-IN-FILE
     REMEMBER-CURRENT-FILE
     FORGET-REMEMBERED-FILE
     REMEMBER-CURRENT-OBJECT))
(START-PUZZLE
     (CLOSE-EYES
     OBJECT-IN-HAND-IN-VIEW
     FIND-COLOR-OF-PIECE
```

```
      PIECE-HAS-STRAIGHT-EDGE
      PUT-OBJECT-IN-PILE
      FORGET-CURRENT-PILE
      DESTROY-A-PILE
      LOOK-AT-FIRST-IN-PILE
      REMEMBER-CURRENT-PILE
      FORGET-REMEMBERED-PILE
      REMEMBER-CURRENT-OBJECT))
(LOOK-AT-FIRST-IN-PILE
      (FORGET-CURRENT-PILE
      DESTROY-A-PILE
      REMEMBER-CURRENT-PILE
      FORGET-REMEMBERED-PILE))
[REPEAT     (WHILE  (LOOK-AT-NEXT-IN-PILE))
            (UNTIL  NIL)
   <<
    [REPEAT     (WHILE  NIL)
                (UNTIL  (PIECE-FITS-IN-PUZZLE))
       <<
        (LOOK-AT-NEXT-IN-PILE
             (CLOSE-EYES
             FIND-COLOR-OF-PIECE
             PIECE-HAS-STRAIGHT-EDGE
             PICK-OBJECT-FROM-PILE
             FORGET-CURRENT-PILE
             DESTROY-A-PILE
             REMEMBER-CURRENT-PILE
             FORGET-REMEMBERED-PILE
             REMEMBER-CURRENT-OBJECT))
       >>
    ]
    [REPEAT     (WHILE  (LOOK-AT-NEXT-IN-PILE
                         PIECE-FITS-IN-PUZZLE))
                (UNTIL  NIL)
       <<
        (PICK-OBJECT-FROM-PILE
             (CLOSE-EYES
             FIND-COLOR-OF-PIECE
             PIECE-HAS-STRAIGHT-EDGE
             PIECE-FITS-IN-PUZZLE
             FORGET-CURRENT-PILE
             DESTROY-A-PILE
             LOOK-AT-NEXT-IN-PILE
             REMEMBER-CURRENT-PILE
             FORGET-REMEMBERED-PILE
             REMEMBER-CURRENT-OBJECT))
        (FIT-PIECE-IN-PUZZLE
             (CLOSE-EYES
             OBJECT-IN-HAND-IN-VIEW
             FIND-COLOR-OF-PIECE
             PIECE-HAS-STRAIGHT-EDGE
             PIECE-FITS-IN-PUZZLE
             PUT-OBJECT-IN-PILE
             FORGET-CURRENT-PILE
             DESTROY-A-PILE
             LOOK-AT-FIRST-IN-PILE
             REMEMBER-CURRENT-PILE
             FORGET-REMEMBERED-PILE
             REMEMBER-CURRENT-OBJECT))
        (PIECE-PUT-IN-PUZZLE
```

```
                (CLOSE-EYES
                OBJECT-IN-HAND-IN-VIEW
                FIND-COLOR-OF-PIECE
                PIECE-HAS-STRAIGHT-EDGE
                PIECE-FITS-IN-PUZZLE
                FIT-PIECE-IN-PUZZLE
                PUT-OBJECT-IN-PILE
                FORGET-CURRENT-PILE
                DESTROY-A-PILE
                LOOK-AT-FIRST-IN-PILE
                REMEMBER-CURRENT-PILE
                FORGET-REMEMBERED-PILE
                REMEMBER-CURRENT-OBJECT))
          (LOOK-AT-FIRST-IN-PILE
                (FORGET-CURRENT-PILE
                DESTROY-A-PILE
                REMEMBER-CURRENT-PILE
                FORGET-REMEMBERED-PILE))
          >>
     ]
    >>
]
(PICK-OBJECT-FROM-PILE
      (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      PIECE-HAS-STRAIGHT-EDGE
      PIECE-FITS-IN-PUZZLE
      FORGET-CURRENT-PILE
      DESTROY-A-PILE
      REMEMBER-CURRENT-PILE
      FORGET-REMEMBERED-PILE
      REMEMBER-CURRENT-OBJECT))
(FIT-PIECE-IN-PUZZLE
      (CLOSE-EYES
      OBJECT-IN-HAND-IN-VIEW
      FIND-COLOR-OF-PIECE
      PIECE-HAS-STRAIGHT-EDGE
      PIECE-FITS-IN-PUZZLE
      PUT-OBJECT-IN-PILE
      FORGET-CURRENT-PILE
      DESTROY-A-PILE
      PILE-IS-EMPTY
      REMEMBER-CURRENT-PILE
      FORGET-REMEMBERED-PILE
      REMEMBER-CURRENT-OBJECT))
(PIECE-PUT-IN-PUZZLE
      (CLOSE-EYES
      OBJECT-IN-HAND-IN-VIEW
      FIND-COLOR-OF-PIECE
      PIECE-HAS-STRAIGHT-EDGE
      PIECE-FITS-IN-PUZZLE
      FIT-PIECE-IN-PUZZLE
      PUT-OBJECT-IN-PILE
      FORGET-CURRENT-PILE
      DESTROY-A-PILE
      REMEMBER-CURRENT-PILE
      FORGET-REMEMBERED-PILE
      PILE-IS-EMPTY
      REMEMBER-CURRENT-OBJECT))
[REPEAT    (WHILE  NIL)
```

```
        (UNTIL  (PUZZLE-IS-FINISHED))
<<
 (FORGET-CURRENT-FILE
      (FILE-IS-EMPTY
      DESTROY-A-FILE
      REMEMBER-CURRENT-FILE
      FORGET-REMEMBERED-FILE))
 (PICK-A-FILE
      (FORGET-REMEMBERED-FILE))
 (LOOK-AT-FIRST-IN-FILE
      (FORGET-CURRENT-FILE
      DESTROY-A-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
      FORGET-REMEMBERED-FILE))
 (REMEMBER-CURRENT-OBJECT
      (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      PICK-OBJECT-FROM-FILE
      FORGET-CURRENT-FILE
      DESTROY-A-FILE
      LOOK-AT-NEXT-IN-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
      FORGET-REMEMBERED-FILE))
 (FIND-COLOR-OF-PIECE
      (CLOSE-EYES
      PICK-OBJECT-FROM-FILE
      FORGET-CURRENT-FILE
      DESTROY-A-FILE
      LOOK-AT-NEXT-IN-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
      FORGET-REMEMBERED-FILE
      REMEMBER-CURRENT-OBJECT
      FORGET-REMEMBERED-OBJECT
      REMEMBERED-OBJECT-IN-VIEW))
 (LOOK-AT-NEXT-IN-FILE
      (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      FORGET-COLOR-OF-PIECE
      PIECE-HAS-CURRENT-COLOR
      PICK-OBJECT-FROM-FILE
      FORGET-CURRENT-FILE
      DESTROY-A-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
      FORGET-REMEMBERED-FILE
      REMEMBER-CURRENT-OBJECT
      FORGET-REMEMBERED-OBJECT
      REMEMBERED-OBJECT-IN-VIEW))
 (REPEAT    (WHILE  NIL)
            (UNTIL  (REMEMBERED-OBJECT-IN-VIEW))
    <<
     (REPEAT    (WHILE  NIL)
                (UNTIL  (PIECE-HAS-CURRENT-COLOR))
        <<
         (LOOK-AT-NEXT-IN-FILE
              (OR.*
              (CLOSE-EYES
```

```
                FIND-COLOR-OF-PIECE
                FORGET-COLOR-OF-PIECE
                PIECE-HAS-CURRENT-COLOR
                PIECE-FITS-IN-PUZZLE
                PICK-OBJECT-FROM-FILE
                FORGET-CURRENT-FILE
                DESTROY-A-FILE
                REMEMBER-CURRENT-FILE
                REMEMBERED-FILE-IS-CURRENT
                FORGET-REMEMBERED-FILE
                REMEMBER-CURRENT-OBJECT
                FORGET-REMEMBERED-OBJECT
                REMEMBERED-OBJECT-IN-VIEW)
               (CLOSE-EYES
                FIND-COLOR-OF-PIECE
                PICK-OBJECT-FROM-FILE
                FORGET-CURRENT-FILE
                DESTROY-A-FILE
                REMEMBER-CURRENT-FILE
                REMEMBERED-FILE-IS-CURRENT
                FORGET-REMEMBERED-FILE
                REMEMBER-CURRENT-OBJECT
                FORGET-REMEMBERED-OBJECT)))

]
[REPEAT     (WHILE   (FORGET-COLOR-OF-PIECE
                      PIECE-HAS-CURRENT-COLOR))
            (UNTIL   (REMEMBERED-OBJECT-IN-VIEW))
   <<
    (PICK-OBJECT-FROM-FILE
         (OR.*
         (CLOSE-EYES
          FIND-COLOR-OF-PIECE
          FORGET-COLOR-OF-PIECE
          PIECE-HAS-CURRENT-COLOR
          PIECE-FITS-IN-PUZZLE
          FORGET-CURRENT-FILE
          DESTROY-A-FILE
          LOOK-AT-NEXT-IN-FILE
          REMEMBER-CURRENT-FILE
          REMEMBERED-FILE-IS-CURRENT
          FORGET-REMEMBERED-FILE
          REMEMBER-CURRENT-OBJECT
          FORGET-REMEMBERED-OBJECT)
         (CLOSE-EYES
          FIND-COLOR-OF-PIECE
          FORGET-COLOR-OF-PIECE
          PIECE-HAS-CURRENT-COLOR
          FORGET-CURRENT-FILE
          DESTROY-A-FILE
          LOOK-AT-NEXT-IN-FILE
          REMEMBER-CURRENT-FILE
          REMEMBERED-FILE-IS-CURRENT
          FORGET-REMEMBERED-FILE
          REMEMBER-CURRENT-OBJECT
          FORGET-REMEMBERED-OBJECT)))
     (FORGET-CURRENT-FILE
          (OR.*
          (CLOSE-EYES
           OBJECT-IN-HAND-IN-VIEW
```

```
        FIND-COLOR-OF-PIECE
        FORGET-COLOR-OF-PIECE
        PIECE-HAS-CURRENT-COLOR
        PIECE-FITS-IN-PUZZLE
        FIT-PIECE-IN-PUZZLE
        PUT-OBJECT-IN-PILE
        DESTROY-A-PILE
        LOOK-AT-FIRST-IN-PILE
        REMEMBER-CURRENT-PILE
        REMEMBERED-PILE-IS-CURRENT
        FORGET-REMEMBERED-PILE
        REMEMBER-CURRENT-OBJECT
        FORGET-REMEMBERED-OBJECT)
       (CLOSE-EYES
        OBJECT-IN-HAND-IN-VIEW
        FIND-COLOR-OF-PIECE
        FORGET-COLOR-OF-PIECE
        PIECE-HAS-CURRENT-COLOR
        PUT-OBJECT-IN-PILE
        DESTROY-A-PILE
        LOOK-AT-FIRST-IN-PILE
        REMEMBER-CURRENT-PILE
        REMEMBERED-PILE-IS-CURRENT
        FORGET-REMEMBERED-PILE
        REMEMBER-CURRENT-OBJECT
        FORGET-REMEMBERED-OBJECT)))
(PICK-A-PILE
     (OR.*
     (CLOSE-EYES
      OBJECT-IN-HAND-IN-VIEW
      FIND-COLOR-OF-PIECE
      FORGET-COLOR-OF-PIECE
      PIECE-HAS-CURRENT-COLOR
      PIECE-FITS-IN-PUZZLE
      FIT-PIECE-IN-PUZZLE
      MAKE-A-PILE
      FORGET-REMEMBERED-PILE
      REMEMBER-CURRENT-OBJECT
      FORGET-REMEMBERED-OBJECT)
     (CLOSE-EYES
      OBJECT-IN-HAND-IN-VIEW
      FIND-COLOR-OF-PIECE
      FORGET-COLOR-OF-PIECE
      PIECE-HAS-CURRENT-COLOR
      MAKE-A-PILE
      FORGET-REMEMBERED-PILE
      REMEMBER-CURRENT-OBJECT
      FORGET-REMEMBERED-OBJECT)))
(PUT-OBJECT-IN-PILE
     (OR.*
     (CLOSE-EYES
      OBJECT-IN-HAND-IN-VIEW
      FIND-COLOR-OF-PIECE
      FORGET-COLOR-OF-PIECE
      PIECE-HAS-CURRENT-COLOR
      FORGET-CURRENT-PILE
      DESTROY-A-PILE
      LOOK-AT-FIRST-IN-PILE
      REMEMBER-CURRENT-PILE
      FORGET-REMEMBERED-PILE
```

```
        REMEMBER-CURRENT-OBJECT
        FORGET-REMEMBERED-OBJECT)
       (CLOSE-EYES
        OBJECT-IN-HAND-IN-VIEW
        FIND-COLOR-OF-PIECE
        FORGET-COLOR-OF-PIECE
        PIECE-HAS-CURRENT-COLOR
        FORGET-CURRENT-PILE
        PILE-IS-EMPTY
        DESTROY-A-PILE
        REMEMBER-CURRENT-PILE
        FORGET-REMEMBERED-PILE
        REMEMBER-CURRENT-OBJECT
        FORGET-REMEMBERED-OBJECT)
       (CLOSE-EYES
        OBJECT-IN-HAND-IN-VIEW
        FIND-COLOR-OF-PIECE
        FORGET-COLOR-OF-PIECE
        PIECE-HAS-CURRENT-COLOR
        PIECE-FITS-IN-PUZZLE
        FIT-PIECE-IN-PUZZLE
        FORGET-CURRENT-PILE
        DESTROY-A-PILE
        LOOK-AT-FIRST-IN-PILE
        REMEMBER-CURRENT-PILE
        FORGET-REMEMBERED-PILE
        REMEMBER-CURRENT-OBJECT
        FORGET-REMEMBERED-OBJECT)))
(FORGET-CURRENT-PILE
       (OR.*
       (CLOSE-EYES
        FIND-COLOR-OF-PIECE
        FORGET-COLOR-OF-PIECE
        PIECE-HAS-CURRENT-COLOR
        DESTROY-A-PILE
        REMEMBER-CURRENT-PILE
        FORGET-REMEMBERED-PILE
        REMEMBER-CURRENT-OBJECT
        FORGET-REMEMBERED-OBJECT)
       (CLOSE-EYES
        FIND-COLOR-OF-PIECE
        FORGET-COLOR-OF-PIECE
        PIECE-HAS-CURRENT-COLOR
        PIECE-FITS-IN-PUZZLE
        DESTROY-A-PILE
        LOOK-AT-FIRST-IN-PILE
        REMEMBER-CURRENT-PILE
        FORGET-REMEMBERED-PILE
        REMEMBER-CURRENT-OBJECT
        FORGET-REMEMBERED-OBJECT)))
(PICK-A-PILE
       (OR.*
       (CLOSE-EYES
        FIND-COLOR-OF-PIECE
        FORGET-COLOR-OF-PIECE
        PIECE-HAS-CURRENT-COLOR
        PIECE-FITS-IN-PUZZLE
        FORGET-REMEMBERED-PILE
        REMEMBER-CURRENT-OBJECT
        FORGET-REMEMBERED-OBJECT)
```

```
                (CLOSE-EYES
                 FIND-COLOR-OF-PIECE
                 FORGET-COLOR-OF-PIECE
                 PIECE-HAS-CURRENT-COLOR
                 FORGET-REMEMBERED-FILE
                 REMEMBER-CURRENT-OBJECT
                 FORGET-REMEMBERED-OBJECT)))
          (LOOK-AT-FIRST-IN-FILE
                (OR.*
                (CLOSE-EYES
                 FIND-COLOR-OF-PIECE
                 FORGET-COLOR-OF-PIECE
                 PIECE-HAS-CURRENT-COLOR
                 PIECE-FITS-IN-PUZZLE
                 FORGET-CURRENT-FILE
                 DESTROY-A-FILE
                 REMEMBER-CURRENT-FILE
                 REMEMBERED-FILE-IS-CURRENT
                 FORGET-REMEMBERED-FILE
                 REMEMBER-CURRENT-OBJECT
                 FORGET-REMEMBERED-OBJECT)
                (CLOSE-EYES
                 FIND-COLOR-OF-PIECE
                 FORGET-COLOR-OF-PIECE
                 PIECE-HAS-CURRENT-COLOR
                 FORGET-CURRENT-FILE
                 DESTROY-A-FILE
                 REMEMBER-CURRENT-FILE
                 REMEMBERED-FILE-IS-CURRENT
                 FORGET-REMEMBERED-FILE
                 REMEMBER-CURRENT-OBJECT
                 FORGET-REMEMBERED-OBJECT)))
          >>
     ]
    >>
]
(FORGET-REMEMBERED-OBJECT
     (OR.*
     (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      FORGET-COLOR-OF-PIECE
      PIECE-HAS-CURRENT-COLOR
      PICK-OBJECT-FROM-FILE
      FORGET-CURRENT-FILE
      DESTROY-A-FILE
      LOOK-AT-NEXT-IN-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
      FORGET-REMEMBERED-FILE
      REMEMBER-CURRENT-OBJECT
      REMEMBERED-OBJECT-IN-VIEW)
     (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      FORGET-COLOR-OF-PIECE
      PIECE-HAS-CURRENT-COLOR
      PICK-OBJECT-FROM-FILE
      FORGET-CURRENT-FILE
      DESTROY-A-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
```

```
       FORGET-REMEMBERED-FILE
       REMEMBER-CURRENT-OBJECT
       REMEMBERED-OBJECT-IN-VIEW)))
(PICK-OBJECT-FROM-FILE
     (OR.*
     (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      FORGET-COLOR-OF-PIECE
      PIECE-HAS-CURRENT-COLOR
      PIECE-FITS-IN-PUZZLE
      FORGET-CURRENT-FILE
      DESTROY-A-FILE
      LOOK-AT-NEXT-IN-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
      FORGET-REMEMBERED-FILE
      REMEMBER-CURRENT-OBJECT)
     (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      FORGET-COLOR-OF-PIECE
      PIECE-HAS-CURRENT-COLOR
      FORGET-CURRENT-FILE
      DESTROY-A-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
      FORGET-REMEMBERED-FILE
      REMEMBER-CURRENT-OBJECT)
     (CLOSE-EYES
      FIND-COLOR-OF-PIECE
      FORGET-COLOR-OF-PIECE
      PIECE-HAS-CURRENT-COLOR
      PIECE-FITS-IN-PUZZLE
      FORGET-CURRENT-FILE
      DESTROY-A-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
      FORGET-REMEMBERED-FILE
      REMEMBER-CURRENT-OBJECT)))
(FORGET-CURRENT-FILE
     (OR.*
     (CLOSE-EYES
      OBJECT-IN-HAND-IN-VIEW
      FIND-COLOR-OF-PIECE
      FORGET-COLOR-OF-PIECE
      PIECE-HAS-CURRENT-COLOR
      PUT-OBJECT-IN-FILE
      DESTROY-A-FILE
      LOOK-AT-FIRST-IN-FILE
      REMEMBER-CURRENT-FILE
      REMEMBERED-FILE-IS-CURRENT
      FORGET-REMEMBERED-FILE
      REMEMBER-CURRENT-OBJECT)
     (CLOSE-EYES
      OBJECT-IN-HAND-IN-VIEW
      FIND-COLOR-OF-PIECE
      FORGET-COLOR-OF-PIECE
      PIECE-HAS-CURRENT-COLOR
      PIECE-FITS-IN-PUZZLE
      FIT-PIECE-IN-PUZZLE
      PUT-OBJECT-IN-FILE
```

```
       DESTROY-A-FILE
       LOOK-AT-FIRST-IN-FILE
       REMEMBER-CURRENT-FILE
       REMEMBERED-FILE-IS-CURRENT
       FORGET-REMEMBERED-FILE
       REMEMBER-CURRENT-OBJECT)
      (CLOSE-EYES
       OBJECT-IN-HAND-IN-VIEW
       FIND-COLOR-OF-PIECE
       FORGET-COLOR-OF-PIECE
       PIECE-HAS-CURRENT-COLOR
       PUT-OBJECT-IN-FILE
       DESTROY-A-FILE
       FILE-IS-EMPTY
       REMEMBER-CURRENT-FILE
       REMEMBERED-FILE-IS-CURRENT
       FORGET-REMEMBERED-FILE
       REMEMBER-CURRENT-OBJECT)))
(PICK-A-FILE
      (OR.*
      (CLOSE-EYES
       OBJECT-IN-HAND-IN-VIEW
       FIND-COLOR-OF-PIECE
       FORGET-COLOR-OF-PIECE
       PIECE-HAS-CURRENT-COLOR
       MAKE-A-FILE
       FORGET-REMEMBERED-FILE
       REMEMBER-CURRENT-OBJECT)
      (CLOSE-EYES
       OBJECT-IN-HAND-IN-VIEW
       FIND-COLOR-OF-PIECE
       FORGET-COLOR-OF-PIECE
       PIECE-HAS-CURRENT-COLOR
       MAKE-A-FILE
       FORGET-REMEMBERED-FILE
       PIECE-FITS-IN-PUZZLE
       FIT-PIECE-IN-PUZZLE
       REMEMBER-CURRENT-OBJECT)))
(PUT-OBJECT-IN-FILE
      (OR.*
      (CLOSE-EYES
       OBJECT-IN-HAND-IN-VIEW
       FIND-COLOR-OF-PIECE
       FORGET-COLOR-OF-PIECE
       PIECE-HAS-CURRENT-COLOR
       FORGET-CURRENT-FILE
       DESTROY-A-FILE
       LOOK-AT-FIRST-IN-FILE
       REMEMBER-CURRENT-FILE
       FORGET-REMEMBERED-FILE
       REMEMBER-CURRENT-OBJECT)
      (CLOSE-EYES
       OBJECT-IN-HAND-IN-VIEW
       FIND-COLOR-OF-PIECE
       FORGET-COLOR-OF-PIECE
       PIECE-HAS-CURRENT-COLOR
       FORGET-CURRENT-FILE
       DESTROY-A-FILE
       PIECE-FITS-IN-PUZZLE
       FIT-PIECE-IN-PUZZLE
```

```
      LOOK-AT-FIRST-IN-FILE
      REMEMBER-CURRENT-FILE
      FORGET-REMEMBERED-FILE
      REMEMBER-CURRENT-OBJECT)))
[REPEAT    (WHILE  (LOOK-AT-FIRST-IN-FILE))
           (UNTIL  (FILE-IS-EMPTY))
   <<
    (LOOK-AT-FIRST-IN-FILE
         (OR.*
         (CLOSE-EYES
          FIND-COLOR-OF-PIECE
          FORGET-COLOR-OF-PIECE
          PIECE-HAS-CURRENT-COLOR
          PIECE-FITS-IN-PUZZLE
          FORGET-CURRENT-FILE
          DESTROY-A-FILE
          REMEMBER-CURRENT-FILE
          FORGET-REMEMBERED-FILE
          REMEMBER-CURRENT-OBJECT)
         (FORGET-CURRENT-FILE
          DESTROY-A-FILE
          REMEMBER-CURRENT-FILE
          FORGET-REMEMBERED-FILE)))
    (PICK-OBJECT-FROM-FILE
         (OR.*
         (CLOSE-EYES
          FIND-COLOR-OF-PIECE
          FORGET-COLOR-OF-PIECE
          PIECE-HAS-CURRENT-COLOR
          PIECE-FITS-IN-PUZZLE
          FORGET-CURRENT-FILE
          DESTROY-A-FILE
          LOOK-AT-NEXT-IN-FILE
          REMEMBER-CURRENT-FILE
          FORGET-REMEMBERED-FILE
          REMEMBER-CURRENT-OBJECT)
         (CLOSE-EYES
          FIND-COLOR-OF-PIECE
          FORGET-COLOR-OF-PIECE
          PIECE-HAS-CURRENT-COLOR
          PIECE-FITS-IN-PUZZLE
          FORGET-CURRENT-FILE
          DESTROY-A-FILE
          REMEMBER-CURRENT-FILE
          FORGET-REMEMBERED-FILE
          REMEMBER-CURRENT-OBJECT)))
    (FIT-PIECE-IN-PUZZLE
         (OR.*
         (CLOSE-EYES
          OBJECT-IN-HAND-IN-VIEW
          FIND-COLOR-OF-PIECE
          FORGET-COLOR-OF-PIECE
          PIECE-HAS-CURRENT-COLOR
          PIECE-FITS-IN-PUZZLE
          PUT-OBJECT-IN-FILE
          FORGET-CURRENT-FILE
          DESTROY-A-FILE
          LOOK-AT-FIRST-IN-FILE
          REMEMBER-CURRENT-FILE
          FORGET-REMEMBERED-FILE
```

```
                REMEMBER-CURRENT-OBJECT)
               (CLOSE-EYES
                OBJECT-IN-HAND-IN-VIEW
                FIND-COLOR-OF-PIECE
                FORGET-COLOR-OF-PIECE
                PIECE-HAS-CURRENT-COLOR
                PIECE-FITS-IN-PUZZLE
                PUT-OBJECT-IN-PILE
                FORGET-CURRENT-PILE
                PILE-IS-EMPTY
                DESTROY-A-PILE
                REMEMBER-CURRENT-PILE
                FORGET-REMEMBERED-PILE
                REMEMBER-CURRENT-OBJECT)))
          (PIECE-PUT-IN-PUZZLE
               (OR.*
               (CLOSE-EYES
                OBJECT-IN-HAND-IN-VIEW
                FIND-COLOR-OF-PIECE
                FORGET-COLOR-OF-PIECE
                PIECE-HAS-CURRENT-COLOR
                PIECE-FITS-IN-PUZZLE
                FIT-PIECE-IN-PUZZLE
                PUT-OBJECT-IN-PILE
                FORGET-CURRENT-PILE
                DESTROY-A-PILE
                LOOK-AT-FIRST-IN-PILE
                REMEMBER-CURRENT-PILE
                FORGET-REMEMBERED-PILE
                REMEMBER-CURRENT-OBJECT)
               (CLOSE-EYES
                OBJECT-IN-HAND-IN-VIEW
                FIND-COLOR-OF-PIECE
                FORGET-COLOR-OF-PIECE
                PIECE-HAS-CURRENT-COLOR
                PIECE-FITS-IN-PUZZLE
                FIT-PIECE-IN-PUZZLE
                PUT-OBJECT-IN-PILE
                FORGET-CURRENT-PILE
                PILE-IS-EMPTY
                DESTROY-A-PILE
                REMEMBER-CURRENT-PILE
                FORGET-REMEMBERED-PILE
                REMEMBER-CURRENT-OBJECT)))
          >>
     ]
     ;
]
(FORGET-CURRENT-PILE
     (PILE-IS-EMPTY
     DESTROY-A-PILE
     REMEMBER-CURRENT-PILE
     PUZZLE-IS-FINISHED
     FORGET-REMEMBERED-PILE))
[REPEAT    (WHILE  (PICK-A-PILE FORGET-REMEMBERED-PILE))
           (UNTIL  (THERE-ARE-NO-PILES))
   <<
    (PICK-A-PILE
         (PUZZLE-IS-FINISHED
         FORGET-REMEMBERED-PILE))
```

```
    (DESTROY-A-FILE
         (PUZZLE-IS-FINISHED
         FORGET-CURRENT-FILE
         FILE-IS-EMPTY
         REMEMBER-CURRENT-FILE
         FORGET-REMEMBERED-FILE))
    >>
]
(PUZZLE-IS-FINISHED
     (THERE-ARE-NO-FILES))
>>
```

APPENDIX 3

FOURTH PUZZLE EXPERIMENT

```
?*** UT LISP - VER 4.1 (78/04/10).
(*** PROSYS SUBSYSTEM 1.1 (79/04/15))
(*** CRAPS CONTROL ***)

(*** EXPERIMENT 4 SPACE SHIP PUZZLE ***)


 INITIAL WORKING MEMORY
((NUMBER-OF-PIECES 24)
 (NUMBER-IN-HEAP 24)
 (NUMBER-OF-PILES 0)
 (IN-PUZZLE NOTHING)
 (CURRENT-COLOR NOTHING)
 (HOLDING NOTHING)
 (LOOKING-AT NOTHING)
 (CURRENT-PILE NONE)
 (REMEMBERED-PILE NONE)
 (REMEMBERED-OBJECT NONE)
 (ALL-PILES)
 (IN-HEAP (PIECE 22 RED))
 (IN-HEAP (PIECE 23 BLACK))
 (IN-HEAP (PIECE 12 BLACK))
 (IN-HEAP (PIECE 13 BLACK))
 (IN-HEAP (PIECE 14 BLACK))
 (IN-HEAP (PIECE 15 BLACK))
 (IN-HEAP (PIECE 24 RED))
 (IN-HEAP (PIECE 25 BLACK))
 (IN-HEAP (PIECE 32 BLACK))
 (IN-HEAP (PIECE 33 RED))
 (IN-HEAP (PIECE 34 SILVER))
 (IN-HEAP (PIECE 35 SILVER))
 (IN-HEAP (PIECE 42 BLACK))
 (IN-HEAP (PIECE 43 BLACK))
 (IN-HEAP (PIECE 44 SILVER))
 (IN-HEAP (PIECE 45 BLACK))
 (IN-HEAP (PIECE 52 BROWN))
 (IN-HEAP (PIECE 53 BROWN))
 (IN-HEAP (PIECE 55 BLACK))
 (IN-HEAP (PIECE 62 BROWN))
 (IN-HEAP (PIECE 63 BROWN))
 (IN-HEAP (PIECE 64 BROWN))
 (IN-HEAP (PIECE 54 BROWN))
 (IN-HEAP (PIECE 65 BLACK))
 (L 22 0 RED)
 (L 23 29 BLACK)
 (T 23 30 BLACK)
 (R 23 -36 BLACK)
 (B 23 -40 BLACK)
 (L 12 0 BLACK)
 (T 12 0 BLACK)
 (R 12 23 BLACK)
 (R 13 -24 BLACK)
 (B 13 -30 BLACK)
 (L 14 24 BLACK)
```

```
(T 14 0 BLACK)
(R 14 31 BLACK)
(R 15 0 BLACK)
(B 15 38 BLACK)
(L 24 36 RED)
(T 24 42 BLACK)
(R 24 -44 BLACK)
(L 65 108 BROWN)
(T 65 104 BLACK)
(B 24 46 RED)
(L 25 44 BLACK)
(T 25 -38 BLACK)
(R 25 0 BLACK)
(B 25 47 BLACK)
(L 32 0 BLACK)
(T 32 -35 BLACK)
(R 32 52 BLACK)
(B 32 -53 BLACK)
(L 33 -52 BLACK)
(T 33 40 RED)
(R 33 54 RED)
(B 33 -55 BLACK)
(L 34 -54 RED)
(T 34 -46 BLACK)
(R 34 56 SILVER)
(B 34 -57 SILVER)
(L 35 -56 SILVER)
(T 35 -47 SILVER)
(R 35 0 BLACK)
(B 35 60 SILVER)
(L 42 0 L-BLACK)
(T 42 53 BLACK)
(R 42 -68 BLACK)
(T 44 57 SILVER)
(L 64 107 BROWN)
(T 64 -103 BROWN)
(R 44 76 SILVER)
(B 44 -77 SILVER)
(L 45 -76 BLACK)
(T 45 -60 BLACK)
(R 45 0 BLACK)
(B 45 82 BLACK)
(L 52 0 BROWN)
(T 52 -69 BLACK)
(R 52 -86 BROWN)
(B 52 101 BROWN)
(L 53 86 BROWN)
(T 53 74 YELLOW)
(B 12 -27 BLACK)
(L 13 -23 BLACK)
(T 13 0 BLACK)
(R 53 87 BROWN)
(B 53 -102 BROWN)
(L 55 88 BLACK)
(T 55 82 BLACK)
(R 55 0 BLACK)
(B 42 69 BLACK)
(L 43 68 BLACK)
(T 43 55 BLACK)
(R 43 -62 SILVER)
```

```
(B 43 -64 BLACK)
(L 44 72 SILVER)
(B 55 -104 BLACK)
(T 22 27 BLACK)
(R 22 -29 RED)
(B 22 35 RED)
(L 62 0 BROWN)
(T 62 -101 BROWN)
(R 62 106 BROWN)
(B 62 0 BROWN)
(L 63 -106 BROWN)
(T 63 102 BROWN)
(R 63 -107 BROWN)
(B 63 0 BROWN)
(R 64 -108 BROWN)
(B 64 0 BROWN)
(B 14 -42 BLACK)
(L 15 -31 BLACK)
(T 15 0 BLACK)
(L 54 -87 BROWN)
(T 54 77 SILVER)
(R 54 -88 BROWN)
(B 54 103 BROWN)
(R 65 0 BLACK)
(B 65 0 BLACK)
```

```
*** BEGIN RUN ***

1. LOOK-AT-PIECE-IN-HEAP
2. PICK-UP-OBJECT-IN-VIEW
3. MAKE-A-FILE
4. LOOK-AT-PIECE-IN-HEAP
5. PICK-UP-OBJECT-IN-VIEW
6. PUT-OBJECT-IN-FILE
7. LOOK-AT-PIECE-IN-HEAP
8. PICK-UP-OBJECT-IN-VIEW
9. PUT-OBJECT-IN-FILE

10. LOOK-AT-PIECE-IN-HEAP
11. PICK-UP-OBJECT-IN-VIEW
12. PUT-OBJECT-IN-FILE
13. LOOK-AT-PIECE-IN-HEAP
14. PICK-UP-OBJECT-IN-VIEW
15. PUT-OBJECT-IN-FILE
16. LOOK-AT-PIECE-IN-HEAP
17. PICK-UP-OBJECT-IN-VIEW
18. PUT-OBJECT-IN-FILE
19. LOOK-AT-PIECE-IN-HEAP

20. PICK-UP-OBJECT-IN-VIEW
21. PUT-OBJECT-IN-FILE
22. LOOK-AT-PIECE-IN-HEAP
23. PICK-UP-OBJECT-IN-VIEW
24. PUT-OBJECT-IN-FILE
25. LOOK-AT-PIECE-IN-HEAP
26. PICK-UP-OBJECT-IN-VIEW
27. PUT-OBJECT-IN-FILE
28. LOOK-AT-PIECE-IN-HEAP
29. PICK-UP-OBJECT-IN-VIEW

30. PUT-OBJECT-IN-FILE
31. LOOK-AT-PIECE-IN-HEAP
32. PICK-UP-OBJECT-IN-VIEW
33. PUT-OBJECT-IN-FILE
34. LOOK-AT-PIECE-IN-HEAP
35. PICK-UP-OBJECT-IN-VIEW
36. PUT-OBJECT-IN-FILE
37. LOOK-AT-PIECE-IN-HEAP
38. PICK-UP-OBJECT-IN-VIEW
39. PUT-OBJECT-IN-FILE

40. LOOK-AT-PIECE-IN-HEAP
41. PICK-UP-OBJECT-IN-VIEW
42. PUT-OBJECT-IN-FILE
43. LOOK-AT-PIECE-IN-HEAP
44. PICK-UP-OBJECT-IN-VIEW
45. PUT-OBJECT-IN-FILE
46. LOOK-AT-PIECE-IN-HEAP
47. PICK-UP-OBJECT-IN-VIEW
48. PUT-OBJECT-IN-FILE
49. LOOK-AT-PIECE-IN-HEAP

50. PICK-UP-OBJECT-IN-VIEW
51. PUT-OBJECT-IN-FILE
52. LOOK-AT-PIECE-IN-HEAP
```

```
    53. PICK-UP-OBJECT-IN-VIEW
    54. PUT-OBJECT-IN-PILE
    55. LOOK-AT-PIECE-IN-HEAP
    56. PICK-UP-OBJECT-IN-VIEW
    57. PUT-OBJECT-IN-PILE
    58. LOOK-AT-PIECE-IN-HEAP
    59. PICK-UP-OBJECT-IN-VIEW

    60. PUT-OBJECT-IN-PILE
    61. LOOK-AT-PIECE-IN-HEAP
    62. PICK-UP-OBJECT-IN-VIEW
    63. PUT-OBJECT-IN-PILE
    64. LOOK-AT-PIECE-IN-HEAP
    65. PICK-UP-OBJECT-IN-VIEW
    66. PUT-OBJECT-IN-PILE
    67. LOOK-AT-PIECE-IN-HEAP
    68. PICK-UP-OBJECT-IN-VIEW
    69. PUT-OBJECT-IN-PILE

    70. LOOK-AT-PIECE-IN-HEAP

      WORKING MEMORY
((NUMBER-OF-PIECES 24)
 (NUMBER-IN-HEAP 1)
 (NUMBER-OF-PILES 1)
 (IN-PUZZLE NOTHING)
 (CURRENT-COLOR NOTHING)
 (HOLDING NOTHING)
 (LOOKING-AT (PIECE 22 RED))
 (CURRENT-PILE 1)
 (REMEMBERED-PILE NONE)
 (REMEMBERED-OBJECT NONE)
 (ALL-PILES 1)
 (IN-HEAP (PIECE 22 RED))
 (PILE 1
       (PIECE 65 BLACK)
       (PIECE 54 BROWN)
       (PIECE 64 BROWN)
       (PIECE 63 BROWN)
       (PIECE 62 BROWN)
       (PIECE 55 BLACK)
       (PIECE 53 BROWN)
       (PIECE 52 BROWN)
       (PIECE 45 BLACK)
       (PIECE 44 SILVER)
       (PIECE 43 BLACK)
       (PIECE 42 BLACK)
       (PIECE 35 SILVER)
       (PIECE 34 SILVER)
       (PIECE 33 RED)
       (PIECE 32 BLACK)
       (PIECE 25 BLACK)
       (PIECE 24 RED)
       (PIECE 15 BLACK)
       (PIECE 14 BLACK)
       (PIECE 13 BLACK)
       (PIECE 12 BLACK)
       (PIECE 23 BLACK)))
    71. PICK-UP-OBJECT-IN-VIEW
    72. PUT-OBJECT-IN-PILE
```

```
    73. EMPTY-HEAP
    74. REMEMBER-CURRENT-FILE
    75. LOOK-AT-FIRST-IN-FILE
    76. REMEMBER-CURRENT-OBJECT
    77. LOOK-AT-NEXT-IN-FILE


**META-RULES-CALLED**

          UNIT
(PICK-OBJECT-FROM-FILE
 (CLOSE-EYES FIND-COLOR-OF-PIECE ...))

          WANT ACTIVE
     PIECE-HAS-STRAIGHT-EDGE

          WANT INACTIVE
     PICK-OBJECT-FROM-FILE

          CURRENTLY ACTIVE
     PICK-OBJECT-FROM-FILE
     LOOK-AT-NEXT-IN-FILE
     REMEMBER-CURRENT-OBJECT
     FIND-COLOR-OF-PIECE
     CLOSE-EYES
     DESTROY-A-FILE
     FORGET-REMEMBERED-OBJECT
     REMEMBERED-FILE-IS-CURRENT
     REMEMBER-CURRENT-FILE
     FORGET-REMEMBERED-FILE
     FORGET-CURRENT-FILE

          TRY LIST
     PICK-OBJECT-FROM-FILE
     LOOK-AT-NEXT-IN-FILE
     LOOK-AT-NEXT-IN-FILE
     PICK-OBJECT-FROM-FILE
     LOOK-AT-NEXT-IN-FILE



    78. LOOK-AT-NEXT-IN-FILE
    79. PICK-OBJECT-FROM-FILE

    80. FORGET-CURRENT-FILE

     WORKING MEMORY
((NUMBER-OF-PIECES 24)
 (NUMBER-OF-FILES 1)
 (IN-PUZZLE NOTHING)
 (CURRENT-COLOR NOTHING)
 (HOLDING (PIECE 64 BROWN))
 (LOOKING-AT (PIECE 64 BROWN))
 (CURRENT-FILE NONE)
 (REMEMBERED-FILE 1)
 (REMEMBERED-OBJECT (PIECE 65 BLACK))
 (ALL-FILES 1)
 (FILE 1
       (PIECE 63 BROWN)
```

```
   (PIECE 62 BROWN)
   (PIECE 55 BLACK)
   (PIECE 53 BROWN)
   (PIECE 52 BROWN)
   (PIECE 45 BLACK)
   (PIECE 44 SILVER)
   (PIECE 43 BLACK)
   (PIECE 42 BLACK)
   (PIECE 35 SILVER)
   (PIECE 34 SILVER)
   (PIECE 33 RED)
   (PIECE 32 BLACK)
   (PIECE 25 BLACK)
   (PIECE 24 RED)
   (PIECE 15 BLACK)
   (PIECE 14 BLACK)
   (PIECE 13 BLACK)
   (PIECE 12 BLACK)
   (PIECE 23 BLACK)
   (PIECE 22 RED)
   (PIECE 65 BLACK)
   (PIECE 54 BROWN)))
81. MAKE-A-FILE
82. FORGET-CURRENT-FILE
83. PICK-A-FILE
84. LOOK-AT-FIRST-IN-FILE
85. PICK-OBJECT-FROM-FILE
86. FORGET-CURRENT-FILE
87. PICK-A-FILE
88. PUT-OBJECT-IN-FILE
89. FORGET-CURRENT-FILE

90. PICK-A-FILE
91. LOOK-AT-FIRST-IN-FILE
92. PICK-OBJECT-FROM-FILE
93. FORGET-CURRENT-FILE
94. PICK-A-FILE
95. PUT-OBJECT-IN-FILE
96. FORGET-CURRENT-FILE
97. PICK-A-FILE
98. LOOK-AT-FIRST-IN-FILE
99. PICK-OBJECT-FROM-FILE

100. FORGET-CURRENT-FILE
101. PICK-A-FILE
102. PUT-OBJECT-IN-FILE
103. FORGET-CURRENT-FILE
104. PICK-A-FILE
105. LOOK-AT-FIRST-IN-FILE
106. LOOK-AT-NEXT-IN-FILE
107. PICK-OBJECT-FROM-FILE
108. FORGET-CURRENT-FILE
109. PICK-A-FILE

110. PUT-OBJECT-IN-FILE
111. FORGET-CURRENT-FILE
112. PICK-A-FILE
113. LOOK-AT-FIRST-IN-FILE
114. PICK-OBJECT-FROM-FILE
115. FORGET-CURRENT-FILE
```

```
116. PICK-A-FILE
117. PUT-OBJECT-IN-FILE
118. FORGET-CURRENT-FILE
119. PICK-A-FILE

120. LOOK-AT-FIRST-IN-FILE
121. LOOK-AT-NEXT-IN-FILE
122. LOOK-AT-NEXT-IN-FILE
123. PICK-OBJECT-FROM-FILE
124. FORGET-CURRENT-FILE
125. PICK-A-FILE
126. PUT-OBJECT-IN-FILE
127. FORGET-CURRENT-FILE
128. PICK-A-FILE
129. LOOK-AT-FIRST-IN-FILE

130. PICK-OBJECT-FROM-FILE
131. FORGET-CURRENT-FILE
132. PICK-A-FILE
133. PUT-OBJECT-IN-FILE
134. FORGET-CURRENT-FILE
135. PICK-A-FILE
136. LOOK-AT-FIRST-IN-FILE
137. LOOK-AT-NEXT-IN-FILE
138. LOOK-AT-NEXT-IN-FILE
139. PICK-OBJECT-FROM-FILE

140. FORGET-CURRENT-FILE
141. PICK-A-FILE
142. PUT-OBJECT-IN-FILE
143. FORGET-CURRENT-FILE
144. PICK-A-FILE
145. LOOK-AT-FIRST-IN-FILE
146. PICK-OBJECT-FROM-FILE
147. FORGET-CURRENT-FILE
148. PICK-A-FILE
149. PUT-OBJECT-IN-FILE

150. FORGET-CURRENT-FILE
151. PICK-A-FILE
152. LOOK-AT-FIRST-IN-FILE
153. LOOK-AT-NEXT-IN-FILE
154. PICK-OBJECT-FROM-FILE
155. FORGET-CURRENT-FILE
156. PICK-A-FILE
157. PUT-OBJECT-IN-FILE
158. FORGET-CURRENT-FILE
159. PICK-A-FILE

160. LOOK-AT-FIRST-IN-FILE
161. PICK-OBJECT-FROM-FILE
162. FORGET-CURRENT-FILE
163. PICK-A-FILE
164. PUT-OBJECT-IN-FILE
165. FORGET-CURRENT-FILE
166. PICK-A-FILE
167. LOOK-AT-FIRST-IN-FILE
168. PICK-OBJECT-FROM-FILE
169. FORGET-CURRENT-FILE
```

```
   170. PICK-A-PILE
   171. PUT-OBJECT-IN-PILE
   172. FORGET-CURRENT-PILE
   173. PICK-A-PILE
   174. LOOK-AT-FIRST-IN-PILE
   175. PICK-OBJECT-FROM-PILE
   176. FORGET-CURRENT-PILE
   177. PICK-A-PILE
   178. PUT-OBJECT-IN-PILE
   179. FORGET-CURRENT-PILE

   180. PICK-A-PILE
   181. LOOK-AT-FIRST-IN-PILE
   182. LOOK-AT-NEXT-IN-PILE
   183. PICK-OBJECT-FROM-PILE
   184. FORGET-CURRENT-PILE
   185. PICK-A-PILE
   186. PUT-OBJECT-IN-PILE
   187. FORGET-CURRENT-PILE
   188. PICK-A-PILE
   189. LOOK-AT-FIRST-IN-PILE

   190. PICK-OBJECT-FROM-PILE
   191. FORGET-CURRENT-PILE
   192. PICK-A-PILE
   193. PUT-OBJECT-IN-PILE
   194. FORGET-REMEMBERED-OBJECT
   195. LOOK-AT-FIRST-IN-PILE
   196. PICK-OBJECT-FROM-PILE
   197. START-PUZZLE
   198. LOOK-AT-FIRST-IN-PILE
   199. PICK-OBJECT-FROM-PILE

   200. FIT-PIECE-IN-PUZZLE

     WORKING MEMORY
((NUMBER-OF-PIECES 24)
 (NUMBER-OF-PILES 2)
 (IN-PUZZLE (PIECE 64 BROWN))
 (CURRENT-COLOR NOTHING)
 (HOLDING (PIECE 63 BROWN))
 (LOOKING-AT (PIECE 63 BROWN))
 (CURRENT-PILE 2)
 (REMEMBERED-PILE 1)
 (REMEMBERED-OBJECT NONE)
 (ALL-PILES 2 1)
 (PILE 1
       (PIECE 54 BROWN)
       (PIECE 53 BROWN)
       (PIECE 44 SILVER)
       (PIECE 43 BLACK)
       (PIECE 34 SILVER)
       (PIECE 33 RED)
       (PIECE 24 RED)
       (PIECE 23 BLACK))
 (PILE 2
       (PIECE 62 BROWN)
       (PIECE 55 BLACK)
       (PIECE 52 BROWN)
       (PIECE 45 BLACK)
```

```
        (PIECE 42 BLACK)
        (PIECE 35 SILVER)
        (PIECE 32 BLACK)
        (PIECE 25 BLACK)
        (PIECE 15 BLACK)
        (PIECE 14 BLACK)
        (PIECE 13 BLACK)
        (PIECE 12 BLACK)
        (PIECE 22 RED)
        (PIECE 65 BLACK))
(NUMBER-IN-PUZZLE 1)
(JOINED ((L 64 107 BROWN) (R 63 -107 BROWN)))
(BEING-PUT-IN-PUZZLE (PIECE 63 BROWN)))

   201. PIECE-PUT-IN-PUZZLE
   202. LOOK-AT-FIRST-IN-FILE
   203. PICK-OBJECT-FROM-FILE
   204. FIT-PIECE-IN-PUZZLE
   205. PIECE-PUT-IN-PUZZLE
   206. LOOK-AT-FIRST-IN-FILE
   207. LOOK-AT-NEXT-IN-FILE
   208. PICK-OBJECT-FROM-FILE
   209. FIT-PIECE-IN-PUZZLE

   210. PIECE-PUT-IN-PUZZLE
   211. LOOK-AT-FIRST-IN-FILE
   212. LOOK-AT-NEXT-IN-FILE
   213. PICK-OBJECT-FROM-FILE
   214. FIT-PIECE-IN-PUZZLE
   215. PIECE-PUT-IN-PUZZLE
   216. LOOK-AT-FIRST-IN-FILE
   217. LOOK-AT-NEXT-IN-FILE
   218. PICK-OBJECT-FROM-FILE
   219. FIT-PIECE-IN-PUZZLE

   220. PIECE-PUT-IN-PUZZLE
   221. LOOK-AT-FIRST-IN-FILE
   222. LOOK-AT-NEXT-IN-FILE
   223. LOOK-AT-NEXT-IN-FILE
   224. LOOK-AT-NEXT-IN-FILE
   225. LOOK-AT-NEXT-IN-FILE
   226. LOOK-AT-NEXT-IN-FILE
   227. PICK-OBJECT-FROM-FILE
   228. FIT-PIECE-IN-PUZZLE
   229. PIECE-PUT-IN-PUZZLE

   230. LOOK-AT-FIRST-IN-FILE
   231. PICK-OBJECT-FROM-FILE
   232. FIT-PIECE-IN-PUZZLE
   233. PIECE-PUT-IN-PUZZLE
   234. LOOK-AT-FIRST-IN-FILE
   235. PICK-OBJECT-FROM-FILE
   236. FIT-PIECE-IN-PUZZLE
   237. PIECE-PUT-IN-PUZZLE
   238. LOOK-AT-FIRST-IN-FILE
   239. LOOK-AT-NEXT-IN-FILE

   240. LOOK-AT-NEXT-IN-FILE
   241. LOOK-AT-NEXT-IN-FILE
   242. LOOK-AT-NEXT-IN-FILE
```

```
    243. LOOK-AT-NEXT-IN-FILE
    244. LOOK-AT-NEXT-IN-FILE
    245. PICK-OBJECT-FROM-FILE
    246. FIT-PIECE-IN-PUZZLE
    247. PIECE-PUT-IN-PUZZLE
    248. LOOK-AT-FIRST-IN-FILE
    249. LOOK-AT-NEXT-IN-FILE

    250. LOOK-AT-NEXT-IN-FILE
    251. LOOK-AT-NEXT-IN-FILE
    252. LOOK-AT-NEXT-IN-FILE
    253. LOOK-AT-NEXT-IN-FILE
    254. PICK-OBJECT-FROM-FILE
    255. FIT-PIECE-IN-PUZZLE
    256. PIECE-PUT-IN-PUZZLE
    257. LOOK-AT-FIRST-IN-FILE
    258. LOOK-AT-NEXT-IN-FILE
    259. LOOK-AT-NEXT-IN-FILE

    260. LOOK-AT-NEXT-IN-FILE
    261. LOOK-AT-NEXT-IN-FILE
    262. PICK-OBJECT-FROM-FILE
    263. FIT-PIECE-IN-PUZZLE
    264. PIECE-PUT-IN-PUZZLE
    265. LOOK-AT-FIRST-IN-FILE
    266. LOOK-AT-NEXT-IN-FILE
    267. LOOK-AT-NEXT-IN-FILE
    268. LOOK-AT-NEXT-IN-FILE
    269. PICK-OBJECT-FROM-FILE

    270. FIT-PIECE-IN-PUZZLE
    271. PIECE-PUT-IN-PUZZLE
    272. LOOK-AT-FIRST-IN-FILE
    273. LOOK-AT-NEXT-IN-FILE
    274. LOOK-AT-NEXT-IN-FILE
    275. PICK-OBJECT-FROM-FILE
    276. FIT-PIECE-IN-PUZZLE
    277. PIECE-PUT-IN-PUZZLE
    278. LOOK-AT-FIRST-IN-FILE
    279. LOOK-AT-NEXT-IN-FILE

    280. PICK-OBJECT-FROM-FILE
    281. FIT-PIECE-IN-PUZZLE
    282. PIECE-PUT-IN-PUZZLE
    283. LOOK-AT-FIRST-IN-FILE
    284. PICK-OBJECT-FROM-FILE
    285. FIT-PIECE-IN-PUZZLE
    286. PIECE-PUT-IN-PUZZLE
    287. FORGET-CURRENT-FILE
    288. PICK-A-FILE
    289. LOOK-AT-FIRST-IN-FILE

    290. REMEMBER-CURRENT-OBJECT

     WORKING MEMORY
((NUMBER-OF-PIECES 24)
 (NUMBER-OF-FILES 2)
 (IN-PUZZLE (PIECE 64 BROWN))
 (IN-PUZZLE (PIECE 63 BROWN))
 (IN-PUZZLE (PIECE 62 BROWN))
```

```
(IN-PUZZLE (PIECE 52 BROWN))
(IN-PUZZLE (PIECE 42 BLACK))
(IN-PUZZLE (PIECE 32 BLACK))
(IN-PUZZLE (PIECE 22 RED))
(IN-PUZZLE (PIECE 65 BLACK))
(IN-PUZZLE (PIECE 55 BLACK))
(IN-PUZZLE (PIECE 12 BLACK))
(IN-PUZZLE (PIECE 13 BLACK))
(IN-PUZZLE (PIECE 14 BLACK))
(IN-PUZZLE (PIECE 15 BLACK))
(IN-PUZZLE (PIECE 25 BLACK))
(IN-PUZZLE (PIECE 35 SILVER))
(IN-PUZZLE (PIECE 45 BLACK))
(CURRENT-COLOR NOTHING)
(HOLDING NOTHING)
(LOOKING-AT (PIECE 54 BROWN))
(CURRENT-PILE 1)
(REMEMBERED-PILE 1)
(REMEMBERED-OBJECT (PIECE 54 BROWN))
(ALL-PILES 1 2)
(PILE 1
      (PIECE 54 BROWN)
      (PIECE 53 BROWN)
      (PIECE 44 SILVER)
      (PIECE 43 BLACK)
      (PIECE 34 SILVER)
      (PIECE 33 RED)
      (PIECE 24 RED)
      (PIECE 23 BLACK))
(PILE 2)
(NUMBER-IN-PUZZLE 16)
(JOINED ((L 64 107 BROWN) (R 63 -107 BROWN))
        ((L 63 -106 BROWN) (R 62 106 BROWN))
        ((T 62 -101 BROWN) (B 52 101 BROWN))
        ((T 52 -69 BLACK) (B 42 69 BLACK))
        ((T 42 53 BLACK) (B 32 -53 BLACK))
        ((T 32 -35 BLACK) (B 22 35 RED))
        ((R 64 -108 BROWN) (L 65 108 BROWN))
        ((T 65 104 BLACK) (B 55 -104 BLACK))
        ((T 22 27 BLACK) (B 12 -27 BLACK))
        ((R 12 23 BLACK) (L 13 -23 BLACK))
        ((R 13 -24 BLACK) (L 14 24 BLACK))
        ((R 14 31 BLACK) (L 15 -31 BLACK))
        ((B 15 38 BLACK) (T 25 -38 BLACK))
        ((B 25 47 BLACK) (T 35 -47 SILVER))
        ((B 35 60 SILVER) (T 45 -60 BLACK))))
   291. FIND-COLOR-OF-PIECE
   292. LOOK-AT-NEXT-IN-PILE
   293. PICK-OBJECT-FROM-PILE
   294. FORGET-CURRENT-PILE
   295. PICK-A-PILE
   296. PUT-OBJECT-IN-PILE
   297. FORGET-CURRENT-PILE
   298. PICK-A-PILE
   299. LOOK-AT-FIRST-IN-PILE

   300. LOOK-AT-NEXT-IN-PILE
   301. LOOK-AT-NEXT-IN-PILE
   302. LOOK-AT-NEXT-IN-PILE
   303. LOOK-AT-NEXT-IN-PILE
```

```
    304. LOOK-AT-NEXT-IN-FILE
    305. LOOK-AT-NEXT-IN-FILE
    306. FORGET-REMEMBERED-OBJECT
    307. PICK-OBJECT-FROM-FILE
    308. FORGET-CURRENT-FILE
    309. PICK-A-FILE

    310. PUT-OBJECT-IN-FILE
    311. LOOK-AT-FIRST-IN-FILE
    312. PICK-OBJECT-FROM-FILE
    313. FIT-PIECE-IN-PUZZLE
    314. PIECE-PUT-IN-PUZZLE
    315. LOOK-AT-FIRST-IN-FILE
    316. PICK-OBJECT-FROM-FILE
    317. FIT-PIECE-IN-PUZZLE
    318. PIECE-PUT-IN-PUZZLE
    319. FORGET-CURRENT-FILE

    320. PICK-A-FILE
    321. LOOK-AT-FIRST-IN-FILE
    322. REMEMBER-CURRENT-OBJECT
    323. FIND-COLOR-OF-PIECE
    324. LOOK-AT-NEXT-IN-FILE
    325. LOOK-AT-NEXT-IN-FILE
    326. PICK-OBJECT-FROM-FILE
    327. FORGET-CURRENT-FILE
    328. PICK-A-FILE
    329. PUT-OBJECT-IN-FILE

    330. FORGET-CURRENT-FILE
    331. PICK-A-FILE
    332. LOOK-AT-FIRST-IN-FILE
    333. LOOK-AT-NEXT-IN-FILE
    334. LOOK-AT-NEXT-IN-FILE
    335. LOOK-AT-NEXT-IN-FILE
    336. FORGET-REMEMBERED-OBJECT
    337. PICK-OBJECT-FROM-FILE
    338. FORGET-CURRENT-FILE
    339. PICK-A-FILE

    340. PUT-OBJECT-IN-FILE

     WORKING MEMORY
((NUMBER-OF-PIECES 24)
 (NUMBER-OF-FILES 2)
 (IN-PUZZLE (PIECE 64 BROWN))
 (IN-PUZZLE (PIECE 63 BROWN))
 (IN-PUZZLE (PIECE 62 BROWN))
 (IN-PUZZLE (PIECE 52 BROWN))
 (IN-PUZZLE (PIECE 42 BLACK))
 (IN-PUZZLE (PIECE 32 BLACK))
 (IN-PUZZLE (PIECE 22 RED))
 (IN-PUZZLE (PIECE 65 BLACK))
 (IN-PUZZLE (PIECE 55 BLACK))
 (IN-PUZZLE (PIECE 12 BLACK))
 (IN-PUZZLE (PIECE 13 BLACK))
 (IN-PUZZLE (PIECE 14 BLACK))
 (IN-PUZZLE (PIECE 15 BLACK))
 (IN-PUZZLE (PIECE 25 BLACK))
 (IN-PUZZLE (PIECE 35 SILVER))
```

```
(IN-PUZZLE (PIECE 45 BLACK))
(IN-PUZZLE (PIECE 53 BROWN))
(IN-PUZZLE (PIECE 54 BROWN))
(CURRENT-COLOR SILVER)
(HOLDING NOTHING)
(LOOKING-AT (PIECE 44 SILVER))
(CURRENT-FILE 2)
(REMEMBERED-FILE 1)
(REMEMBERED-OBJECT NONE)
(ALL-FILES 2 1)
(FILE 1 (PIECE 43 BLACK) (PIECE 33 RED)
        (PIECE 24 RED) (PIECE 23 BLACK))
(FILE 2 (PIECE 34 SILVER) (PIECE 44 SILVER))
(NUMBER-IN-PUZZLE 18)
(JOINED ((L 64 107 BROWN) (R 63 -107 BROWN))
        ((L 63 -106 BROWN) (R 62 106 BROWN))
        ((T 62 -101 BROWN) (B 52 101 BROWN))
        ((T 52 -69 BLACK) (B 42 69 BLACK))
        ((T 42 53 BLACK) (B 32 -53 BLACK))
        ((T 32 -35 BLACK) (B 22 35 RED))
        ((R 64 -108 BROWN) (L 65 108 BROWN))
        ((T 65 104 BLACK) (B 55 -104 BLACK))
        ((T 22 27 BLACK) (B 12 -27 BLACK))
        ((R 12 23 BLACK) (L 13 -23 BLACK))
        ((R 13 -24 BLACK) (L 14 24 BLACK))
        ((R 14 31 BLACK) (L 15 -31 BLACK))
        ((B 15 38 BLACK) (T 25 -38 BLACK))
        ((B 25 47 BLACK) (T 35 -47 SILVER))
        ((B 35 60 SILVER) (T 45 -60 BLACK))
        ((R 52 -86 BROWN) (L 53 86 BROWN))
        ((T 63 102 BROWN) (B 53 -102 BROWN))
        ((R 53 87 BROWN) (L 54 -87 BROWN))
        ((T 64 -103 BROWN) (B 54 103 BROWN))
        ((L 55 88 BLACK) (R 54 -88 BROWN))))
   341. LOOK-AT-FIRST-IN-FILE
   342. PICK-OBJECT-FROM-FILE
   343. FIT-PIECE-IN-PUZZLE
   344. PIECE-PUT-IN-PUZZLE
   345. LOOK-AT-FIRST-IN-FILE
   346. PICK-OBJECT-FROM-FILE
   347. FIT-PIECE-IN-PUZZLE
   348. PIECE-PUT-IN-PUZZLE
   349. FORGET-CURRENT-FILE

   350. PICK-A-FILE
   351. LOOK-AT-FIRST-IN-FILE
   352. REMEMBER-CURRENT-OBJECT
   353. FIND-COLOR-OF-PIECE
   354. LOOK-AT-NEXT-IN-FILE
   355. LOOK-AT-NEXT-IN-FILE
   356. LOOK-AT-NEXT-IN-FILE
   357. PICK-OBJECT-FROM-FILE
   358. FORGET-CURRENT-FILE
   359. PICK-A-FILE

   360. PUT-OBJECT-IN-FILE
   361. FORGET-CURRENT-FILE
   362. PICK-A-FILE
   363. LOOK-AT-FIRST-IN-FILE
   364. FORGET-REMEMBERED-OBJECT
```

```
    365. PICK-OBJECT-FROM-FILE
    366. FORGET-CURRENT-FILE
    367. PICK-A-FILE
    368. PUT-OBJECT-IN-FILE
    369. LOOK-AT-FIRST-IN-FILE

    370. PICK-OBJECT-FROM-FILE
    371. FIT-PIECE-IN-PUZZLE
    372. PIECE-PUT-IN-PUZZLE
    373. LOOK-AT-FIRST-IN-FILE
    374. PICK-OBJECT-FROM-FILE
    375. FIT-PIECE-IN-PUZZLE
    376. PIECE-PUT-IN-PUZZLE
    377. FORGET-CURRENT-FILE
    378. PICK-A-FILE
    379. LOOK-AT-FIRST-IN-FILE

    380. REMEMBER-CURRENT-OBJECT
    381. FIND-COLOR-OF-PIECE
    382. LOOK-AT-NEXT-IN-FILE
    383. PICK-OBJECT-FROM-FILE
    384. FORGET-CURRENT-FILE
    385. PICK-A-FILE
    386. PUT-OBJECT-IN-FILE
    387. FORGET-CURRENT-FILE
    388. PICK-A-FILE
    389. LOOK-AT-FIRST-IN-FILE

    390. FORGET-REMEMBERED-OBJECT
    391. PICK-OBJECT-FROM-FILE
    392. FORGET-CURRENT-FILE
    393. PICK-A-FILE
    394. PUT-OBJECT-IN-FILE
    395. LOOK-AT-FIRST-IN-FILE
    396. PICK-OBJECT-FROM-FILE
    397. FIT-PIECE-IN-PUZZLE
    398. PIECE-PUT-IN-PUZZLE
    399. LOOK-AT-FIRST-IN-FILE

    400. PICK-OBJECT-FROM-FILE
    401. FIT-PIECE-IN-PUZZLE

     WORKING MEMORY
((NUMBER-OF-PIECES 24)
 (NUMBER-OF-FILES 2)
 (IN-PUZZLE (PIECE 64 BROWN))
 (IN-PUZZLE (PIECE 63 BROWN))
 (IN-PUZZLE (PIECE 62 BROWN))
 (IN-PUZZLE (PIECE 52 BROWN))
 (IN-PUZZLE (PIECE 42 BLACK))
 (IN-PUZZLE (PIECE 32 BLACK))
 (IN-PUZZLE (PIECE 22 RED))
 (IN-PUZZLE (PIECE 65 BLACK))
 (IN-PUZZLE (PIECE 55 BLACK))
 (IN-PUZZLE (PIECE 12 BLACK))
 (IN-PUZZLE (PIECE 13 BLACK))
 (IN-PUZZLE (PIECE 14 BLACK))
 (IN-PUZZLE (PIECE 15 BLACK))
 (IN-PUZZLE (PIECE 25 BLACK))
 (IN-PUZZLE (PIECE 35 SILVER))
```

```
(IN-PUZZLE (PIECE 45 BLACK))
(IN-PUZZLE (PIECE 53 BROWN))
(IN-PUZZLE (PIECE 54 BROWN))
(IN-PUZZLE (PIECE 34 SILVER))
(IN-PUZZLE (PIECE 44 SILVER))
(IN-PUZZLE (PIECE 23 BLACK))
(IN-PUZZLE (PIECE 43 BLACK))
(IN-PUZZLE (PIECE 24 RED))
(CURRENT-COLOR RED)
(HOLDING (PIECE 33 RED))
(LOOKING-AT (PIECE 33 RED))
(CURRENT-PILE 2)
(REMEMBERED-PILE 1)
(REMEMBERED-OBJECT NONE)
(ALL-PILES 2 1)
(PILE 1)
(PILE 2)
(NUMBER-IN-PUZZLE 23)
(JOINED ((L 64 107 BROWN) (R 63 -107 BROWN))
        ((L 63 -106 BROWN) (R 62 106 BROWN))
        ((T 62 -101 BROWN) (B 52 101 BROWN))
        ((T 52 -69 BLACK) (B 42 69 BLACK))
        ((T 42 53 BLACK) (B 32 -53 BLACK))
        ((T 32 -35 BLACK) (B 22 35 RED))
        ((R 64 -108 BROWN) (L 65 108 BROWN))
        ((T 65 104 BLACK) (B 55 -104 BLACK))
        ((T 22 27 BLACK) (B 12 -27 BLACK))
        ((R 12 23 BLACK) (L 13 -23 BLACK))
        ((R 13 -24 BLACK) (L 14 24 BLACK))
        ((R 14 31 BLACK) (L 15 -31 BLACK))
        ((B 15 38 BLACK) (T 25 -38 BLACK))
        ((B 25 47 BLACK) (T 35 -47 SILVER))
        ((B 35 60 SILVER) (T 45 -60 BLACK))
        ((R 52 -86 BROWN) (L 53 86 BROWN))
        ((T 63 102 BROWN) (B 53 -102 BROWN))
        ((R 53 87 BROWN) (L 54 -87 BROWN))
        ((T 64 -103 BROWN) (B 54 103 BROWN))
        ((L 55 88 BLACK) (R 54 -88 BROWN))
        ((L 35 -56 SILVER) (R 34 56 SILVER))
        ((B 34 -57 SILVER) (T 44 57 SILVER))
        ((T 54 77 SILVER) (B 44 -77 SILVER))
        ((L 45 -76 BLACK) (R 44 76 SILVER))
        ((B 13 -30 BLACK) (T 23 30 BLACK))
        ((R 22 -29 RED) (L 23 29 BLACK))
        ((R 42 -68 BLACK) (L 43 68 BLACK))
        ((B 14 -42 BLACK) (T 24 42 BLACK))
        ((R 23 -36 BLACK) (L 24 36 RED))
        ((T 34 -46 BLACK) (B 24 46 RED))
        ((L 25 44 BLACK) (R 24 -44 BLACK))))

   402. PIECE-PUT-IN-PUZZLE
   403. FORGET-CURRENT-PILE
   404. PICK-A-PILE
   405. DESTROY-A-PILE
   406. PICK-A-PILE
   407. DESTROY-A-PILE
   408. PUZZLE-IS-FINISHED

(END -- EXPLICIT HALT)
(1 -- META RULE CALLS)
```

```
33 PRODUCTIONS IN SYSTEM
127/300 NODES
408 PRODUCTIONS-FIRED
3642 PRODUCTIONS-INSTANTIATED
1831 WM-TRANSACTIONS(68762 NODE ACTIVATIONS)
(68035 TESTS PERFORMED)
(131 MAXIMUM WM SIZE)
(101.62990 MEAN WM SIZE)
(33 MAXIMUM CS SIZE)
(12.96324 MEAN CS SIZE)
(177 MAXIMUM NUMBER TOKENS STORED)
(117.18382 MEAN NUMBER TOKENS STORED)

11.5 MINUTES CPU
```

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REPORT DOCUMENTATION PAGE

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ABSTRACT: During the last few years a number of relatively effective AI programs have been written incorporating considerable amounts of problem specific knowledge. Consequently, the problem of encoding such knowledge in a useful form has emerged as one of the central problems of AI. In particular, declarative representations have attracted much attention partly because of the relative ease with which knowledge can be communicated in this form. Unfortunately, straightforward implementation of declaratively specified knowledge corresponds to a nondeterministic program which incurs enormous computational costs.

This thesis investigates one way to limit this cost. The approach we take is to develop control heuristics for a family of problems from traces of sample solutions generated during a training session with a human expert. Algorithms are presented which recognize a set of patterns in the sequence of 'knowledge applications' and which compile descriptions of these patterns in a control language, called CRAPS. More specifically, patterns of repeating, parallel and common sequences are considered in the analysis. The analysis also produces a set of meta-rules which aid the CRAPS description in the event the sequencing it specifies is inappropriate. The CRAPS description and meta-rules are then used for guidance in solving subsequent problems.

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